$\Sigma \quad W(N_1, N_2, N_n \partial N, \Sigma_j N_j \quad \partial u, \Sigma_{j=1}^{n} E_j N_j$

U

$dU = TdS - PdV$

S, V

$$\left(\frac{\partial U}{\partial S}\right)_V = T \qquad \left(\frac{\partial U}{\partial V}\right)_S = -P \qquad -\left(\frac{\partial P}{\partial S}\right)_V = \left(\frac{\partial T}{\partial V}\right)_S$$

$H = U + PV$

$dH = dU + d(PV)$

$= TdS - PdV + PdV + VdP$

$= TdS + VdP$

S, P

$$\left(\frac{\partial H}{\partial S}\right)_P = T \qquad \left(\frac{\partial H}{\partial P}\right)_S = V \qquad \left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$F = U - TS$

$dF = dU - d(TS)$

$= TdS - PdV - TdS - SdT$

$= -SdT - PdV$

T, V

$$\left(\frac{\partial F}{\partial T}\right)_V = -S \qquad \left(\frac{\partial F}{\partial V}\right)_T = -P \qquad \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

$G = U - TS + VP$

$dG = dU - d(TS) + d(VP)$

$= \cancel{TdS} - \cancel{PdV} - \cancel{TdS} - SdT + VdP + \cancel{PdV}$

$= VdP - SdT$

P, T

$$\left(\frac{\partial G}{\partial P}\right)_T = V \qquad \left(\frac{\partial G}{\partial T}\right)_P = -S \qquad \left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$$

$$\langle xy\rangle = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\, xy\, P(x,y)$$

$$= \int_{-\infty}^{\infty} \underset{\substack{\uparrow \\ \text{odd}}}{x}\, \underset{\substack{\uparrow \\ \text{even}}}{P(x)}\, dx \int y\, P(y)\, dy$$

$$= 0 \cdot 0 = \mathbf{0}$$

$$\langle x+y\rangle = \langle x\rangle + \langle y\rangle = 0+0=0$$

$$\left\langle (x+y)^2 \right\rangle = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\, (x^2+2xy+y^2)$$

$$= \langle x^2\rangle + \langle y^2\rangle \leftarrow \sigma^2$$

$$= a^2 + a^2$$

$$= 2a^2$$

$$\langle x^2 y\rangle = \underset{\substack{\uparrow \\ \neq 0 \\ \neq \infty}}{\langle x^2\rangle}\, \underset{\substack{\uparrow \\ 0}}{\langle y\rangle} = 0$$

$$\mathrm{cov}(x,y) = \mathbf{0}$$

$$\langle x^0 \rangle = \int_{-\infty}^{\infty} x^0 P(x) = \int_{-\infty}^{\infty} P(x) = \frac{1}{\sqrt{2\pi a}} \int_{-\infty}^{\infty} \exp\left(\frac{-x^2}{2a^2}\right)$$

normal dist
1 → b
2 → a
3 → h
4 → h
5 → e

Gaussian dist

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\langle x \rangle)^2}{2\sigma^2}}$$

$$y = \frac{x}{\sqrt{2a^2}}$$
$$dy = \frac{dx}{\sqrt{2a^2}}$$

$$\langle x^0 \rangle = \frac{1}{\sqrt{2\pi a^2}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2a^2}} dx$$

$$= \frac{\sqrt{2a^2}}{\sqrt{2\pi a^2}} \int_{-\infty}^{\infty} e^{-y^2} dy$$

$$= \frac{\sqrt{2a^2}}{\sqrt{2\pi a^2}} \sqrt{\pi} = 1$$

$$\langle x^1 \rangle = \int_{-\infty}^{\infty} dx \, \underset{\substack{\uparrow \\ \text{odd}}}{x} \, \underset{\substack{\uparrow \\ \text{even}}}{P(x)} = 0$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 \, p(x) dx = \frac{1}{\sqrt{2\pi a^2}} \int x^2 e^{-x^2/2a^2} dx$$

$$\uparrow \quad b = \frac{1}{2a^2}$$

$$= \frac{1}{\sqrt{2\pi a^2}} \int_{-\infty}^{\infty} \underbrace{x^2 e^{-bx^2}}_{-\frac{\partial}{\partial b} e^{-bx^2}} dx$$

$$= \frac{1}{\sqrt{2\pi a^2}} \frac{d}{db} \int_{-\infty}^{\infty} dx \, e^{-bx^2}$$

$$= \frac{-1}{\sqrt{2\pi a^2}} \frac{d}{db} \frac{1}{\sqrt{b}} \int_{-\infty}^{\infty} \boxed{dy e^{-y^2}} \to \sqrt{\pi}$$

$$= \frac{1}{\sqrt{2\pi a^2}} \cdot - \frac{d}{db} \frac{\pi^{\frac{1}{2}}}{b^{\frac{1}{2}}}$$

$$= \frac{1}{\sqrt{2\pi a^2}} \cdot + \frac{\pi^{\frac{1}{2}}}{2b^{\frac{3}{2}}} \quad ?$$

$$= \frac{1}{\sqrt{2\pi a^2}} \frac{\pi^{\frac{1}{2}}}{4b^{\frac{3}{2}}}$$

$$= \frac{\pi^{\frac{1}{2}}}{\sqrt{2\pi a^2} \, (2a^2)^{-\frac{3}{2}}} \quad ?$$

$$= \frac{1}{(2a^2)^{\frac{1}{2}}} \frac{1}{2} (2a^2)^{\frac{3}{2}} = a^2$$

$$S = \frac{k_B \ln M^N}{N \ln N - N} = \frac{k_B N \ln M}{N(\ln N - 1)} = \frac{k_B \ln M}{\ln N - 1}$$

$$\overset{N\to\infty}{\approx} \frac{k_B \ln \cancel{M} M}{\ln N} = \frac{k_B}{\ln N} \ln M$$

$$= \frac{k_B}{\ln N} \ln \frac{V}{v}$$

$$\frac{\partial S}{\partial V} = \frac{k_B}{\ln N} \frac{1}{v} \frac{v}{V} = \frac{k_B}{V \ln N} = p T^{-1}$$

$$pV = k_B T$$

$$k_B N \frac{\ln M}{N \ln N - N}$$

$$S = k_B \ln W_{MB} = k_B \ln \frac{1}{N!} M^N$$

$$dU = T\,dS - P\,dV + \mu\,dn$$

$$dS = \frac{1}{T}\,dU + \frac{P}{T}\,dV - \frac{\mu}{T}\,dn$$

Constant $N, U \Rightarrow \left(\frac{\partial S}{\partial V}\right)_{N,U} = \frac{P}{T}$

as $dU = dn = 0$

$$S = k_B \ln M^N (N \ln N - N)$$

$$= k_B N^2 \ln M (\ln N - 1)$$

$$= k_B N^2 (\ln N \ln M - \ln M)$$

goal: $\boxed{pV = N k_B T}$

$$= k_B N^2 (\ln (M^N) - \ln (M))$$

$$= k_B N^2 \left(\ln \left(\frac{V}{v}\right)^N - \ln \left(\frac{V}{v}\right)\right)$$

$\hookrightarrow \ln \left(\frac{V}{v}\right)^{N-1}$

$$\frac{\partial S}{\partial V} = k_B N^2 \frac{1}{v} \left(\frac{V}{v}\right)^{N-2} \left(\frac{v}{V}\right)^{N-1}$$

$$= k_B N^2 \frac{1}{v} \frac{v}{V} = k_B N^2 \frac{1}{V}$$

$$dV = \frac{T}{p} dS$$

$$V = \frac{T}{p} S$$

$$pV = TS$$

$$pV = k_B T \ln M^N (N \ln N - N) = k_B T (N \ln M)(N \ln N - N)$$

$$= k_B T N (\ln (M) \ln (N) -$$

$$p = \frac{W}{\Omega}$$

$$S = k_B \ln \frac{1}{N!} M^N$$

$$= k_B \left(\ln \frac{1}{N!} + \ln M^N \right)$$

$$= k_B \left(N \ln M - \ln N! \right)$$

$$= k_B \left(N \ln M - N \ln N + N \right)$$

$$= k_B N \left(\ln \frac{M}{N} + 1 \right) = k_B N \left(\ln \frac{V}{N v} + 1 \right)$$

$$\frac{\partial S}{\partial V} = k_B N \frac{1}{N v} \frac{N v}{V} = \underbrace{\frac{k_B N}{V} = \frac{p}{T}}_{pV = N k_B T}$$

ya in $ay+b$

$$P(x) = \int_0^\infty P(x,y)\,dy = \left[\frac{1}{\ell^2} \cdot e^{\frac{-x+y}{\ell}} \cdot \frac{1}{\left(-\frac{1}{\ell}\right)}\right]_0^\infty$$

$$= -\frac{1}{\ell}\left[e^{-\frac{x+y}{\ell}}\right]_0^\infty$$

$$= -\frac{1}{\ell}\left(\left(e^{-\frac{x}{\ell}-\frac{\infty}{\ell}}\right) - \left(e^{-\frac{x}{\ell}-\frac{0}{\ell}}\right)\right)$$

$$= -\frac{1}{\ell}\left(0 - e^{-\frac{x}{\ell}}\right)$$

$$= \frac{1}{\ell} e^{-\frac{x}{\ell}}$$

$$E(x^0) = \int x^0 P(x)\,dx = \int_0^\infty P(x)\,dx = \frac{1}{\ell}\int_0^\infty e^{-\frac{x}{\ell}} = \frac{1}{\ell} \cdot \left(\frac{1}{\left(-\frac{1}{\ell}\right)}\right)\left[e^{-\frac{x}{\ell}}\right]_0^\infty$$

$$= \frac{1}{\ell} \cdot -\ell\left(e^{-\frac{\infty}{\ell}} - e^{-\frac{0}{\ell}}\right)$$

$$= -1 \cdot -e^0$$

$$= 1$$

$$E(x) = \int_0^\infty x P(x)\,dx = -\frac{\ell x}{\ell} e^{-\frac{x}{\ell}}\Big|_0^\infty - \int_0 -\frac{\ell}{\ell} e^{-\frac{x}{\ell}}$$

$$= 0 - 0 + \cancel{\ell}\left[\frac{1}{\left(-\frac{1}{\ell}\right)} e^{-\frac{x}{\ell}}\right]_0^\infty$$

$$= -\ell^{\cancel{x}}\left(e^{-\frac{\infty}{\ell}} - e^{-\frac{0}{\ell}}\right)$$

$$= -\ell^{\cancel{x}} \cdot - e^0$$

$$= \ell^{\cancel{x}}$$

$P \; dV$

$P = x \qquad dV = \frac{1}{\ell}e^{-\frac{x}{\ell}}$

$dP = 1 \qquad V = -e^{-\frac{x}{\ell}}$

$$P(y) = \frac{1}{\ell} e^{-\frac{y}{\ell}}$$

$$E(y^2) = \int_0^\infty y^2 \frac{1}{\ell} e^{-\frac{y}{\ell}} dy = -2y e^{-\frac{y}{\ell}}\Big|_0^\infty + \int_0^\infty 2y e^{-\frac{y}{\ell}} dy$$

$$= \int_0^\infty 2y e^{-\frac{y}{\ell}} dy$$

$$= -2\ell y e^{-\frac{y}{\ell}}\Big|_0^\infty + \int_0^\infty 2\ell e^{-\frac{y}{\ell}} dy$$

$$= 2\ell \cdot -\ell \left[e^{-\frac{y}{\ell}}\right]_0^\infty$$

$$= -2\ell^2\left(e^{-\frac{\infty}{\ell}} - e^{-\frac{0}{\ell}}\right)$$

$$= +2\ell^2$$

$P \; dV$

$P = y^2 \qquad dV = \frac{1}{\ell}e^{-\frac{y}{\ell}}$

$dP = 2y \qquad V = -e^{-\frac{y}{\ell}}$

$P = 2y \qquad dV = e^{-\frac{y}{\ell}}$

$dP = 2 \qquad V = -\ell e^{-\frac{y}{\ell}}$

2a

non-degenerate → $g=1$

$$Z = \sum_{j=1}^{n} g_j \, e^{-\frac{E_j}{k_B T}}$$

$$= e^{+\frac{\mu_B B}{k_B T}} + e^{-\frac{\mu_B B}{k_B T}} = 2 \cdot \frac{1}{2}\left(e^{\frac{\mu_B B}{k_B T}} + e^{-\frac{\mu_B B}{k_B T}}\right)$$

$$= 2\cosh\left(\mu_B B / k_B T\right)$$

$$p = \frac{N_j}{N} = \frac{g_j}{Z} e^{-\frac{E_j}{k_B T}}$$

$$\frac{p_2}{p_1} = \frac{g_2}{g_1} \frac{e^{-\frac{E_2}{k_B T}}}{e^{-\frac{E_1}{k_B T}}} = \frac{e^{\frac{E_1}{k_B T}}}{e^{\frac{E_2}{k_B T}}} = \frac{e^{\frac{-\mu_B B}{k_B T}}}{e^{\frac{\mu_B B}{k_B T}}} = \frac{1}{e^{\frac{2\mu_B B}{k_B T}}}$$

$$= e^{-2\mu_B B / k_B T}$$

$$\langle E \rangle = \sum_{j=1}^{2} p_j E_j = \frac{1}{Z}\left(e^{-\frac{E_2}{k_B T}} E_2 + e^{-\frac{E_1}{k_B T}} E_1\right)$$

$$= \frac{1}{Z}\left(e^{-\frac{\mu_B B}{k_B T}} \mu_B B - e^{\frac{\mu_B B}{k_B T}} \mu_B B\right)$$

$$= \frac{-2\mu_B B \sinh\left(e^{\frac{\mu_B B}{k_B T}}\right)}{2\cosh\left(e^{\frac{\mu_B B}{k_B T}}\right)}$$

$$= -\mu_B B \tanh e^{\frac{\mu_B B}{k_B T}}$$

binomial distribution

$$P(N_1) = \binom{N}{N_1} p_1^{N_1} p_2^{N-N_1} = \binom{N}{N_1} p_1^{N_1} p_2^{N_2}$$

Name:
Student Number:

$$P(x) = \int_0^\infty P(x,y)\,dy = \int_0^l \frac{1}{l^2}\,dy = \int_0^l l^{-2}\,dy$$

$$= \frac{1}{l^2}\int_0^l dy = \frac{l}{l^2} = \frac{1}{l}$$

$$E(x^0) = \int_0^l \frac{1}{l}\,dx = \frac{l}{l} = 1 \Rightarrow \text{yes}$$

$$E(xy) = \int_0^l\int_0^l xy\,P(x,y)\,dx\,dy = \int_0^l\int_0^l xy\frac{1}{l^2}\,dx\,dy$$

$$= \int_0^l x\,\frac{1}{2}y^2\frac{1}{l^2}\,dx = \frac{\frac{1}{2}y^2}{l^2}\int_0^l x\,dx = \left[\frac{\frac{1}{4}x^2y^2}{l^2}\right]_0^l$$

$$\frac{1}{4}l^2 - 0 = \frac{1}{4}l^2$$

$$E(y^2) = \int_0^l y^2 p(y)\,dy = \int_0^l y^2\frac{1}{l^{\cancel{2}}}\,dy = \frac{1}{3l}\left[y^3\right]_0^l = \frac{1}{3l}l^3 = \frac{1}{3}l^2$$

$$E(y) = \int_0^l y\,P(y)\,dy = \frac{1}{l}\left[\frac{1}{2}y^2\right]_0^l = \frac{1}{2}l$$

$$\sigma_y = \sqrt{E(y^2) - E(y)^2} = \sqrt{\frac{1}{3}l^2 - \frac{1}{4}l^2} = \sqrt{\frac{1}{12}l^2} = \frac{l}{\sqrt{12}}$$

$$\rightarrow \frac{l}{\sqrt{12}} = \frac{l}{\sqrt{4\cdot 3}} = \frac{l}{\sqrt{4}\sqrt{3}} = \frac{l}{2\sqrt{3}}$$

Undetermined / Lagrange multipliers

$$\mathcal{L} = S + \overset{\downarrow}{\alpha}\left(\sum_{j=1}^{n} N_j - N\right) + \overset{\downarrow}{\beta}\left(\sum_{j=1}^{n} N_j E_j - U\right)$$

$\downarrow$ maximize $\mathcal{L} \Rightarrow \dfrac{\partial \mathcal{L}}{\partial N_i} = 0$

$$\frac{\partial}{\partial N_i} S_{FD} + \alpha \frac{\partial}{\partial N_i}\left(\sum_{j=1}^{n} \underset{\substack{\downarrow \\ 1}}{N_j} - \underset{\substack{\downarrow \\ 0}}{N}\right) + \beta \frac{\partial}{\partial N_i}\left(\sum_{j=1}^{n} \underset{\substack{\downarrow \\ E_i}}{N_j E_j} - \underset{\substack{\downarrow \\ 0}}{U}\right) = 0$$

$$\frac{\partial S_{FD}}{\partial N_i} + \alpha + \beta E_i = 0$$

$$\frac{S_{FD}}{K_B} = \ln\left(\prod_{j=1}^{n} \frac{g_j!}{N_j!\,(g_j - N_j)!}\right) = \sum_{j=1}^{n} \ln \frac{g_j!}{N_j!\,(g_j - N_j)!}$$

$$= \sum_{j=1}^{n}\left(\ln g_j! - \ln N_j! - \ln (g_j - N_j)!\right)$$

$$= \sum_{j=1}^{n}\left(g_j \ln g_j - g_j - N_j \ln N_j + N_j + (g_j - N_j) - (g_j - N_j)\ln(g_j - N_j)\right)$$

$$= \sum_{j=1}^{N}\left(g_j \ln g_j - N_j \ln N_j - (g_j - N_j)\ln(g_j - N_j)\right.$$

$$\frac{dS_{FD}}{dN_i} = k_B\left(-\ln N_i - N_i\frac{1}{N_i} + \ln(g_i - N_i) - \frac{(g_i - N_i)}{g_i - N_i}\right) \cdot\cdot -1$$

$$= k_B\left(-\ln N_i + \ln(g_i - N_i)\right)$$

$$= k_B \ln\left(\frac{g_i - N_i}{N_i}\right)$$

$$k_B \ln\frac{g_i - N_i}{N_i} + \alpha + \beta E_i = 0$$

$$\frac{g_i}{N_i} - 1 = e^{-\frac{\alpha}{k_B} - \frac{\beta}{k_B} E_i}$$

---

most probable distribution

$$\boxed{\frac{g_i}{N_i} = 1 + e^{-\frac{\alpha}{k_B} - \frac{\beta}{k_B} E_i}}$$

for Boltzmann statistics

$$\frac{g_j}{N_j} = e^{-\frac{\alpha + \beta E_j}{k_B}}$$

$$\boxed{S_{FD} = -\alpha N - \beta U + k_B \sum_{j=1}^{n} g_j \ln \left(1 + e^{\frac{\alpha+\beta E_j}{k_B}}\right)}$$

$$= k_B \sum_{j=1}^{n} \left(g_j \ln g_j - N_j \ln N_j - (g_j - N_j) \ln (g_j - N_j)\right)$$

$$= k_B \sum_{j=1}^{n} \left(g_j \ln \frac{g_j}{g_j - N_j} - N_j \underbrace{\ln \frac{N_j}{g_j - N_j}}_{\ln \frac{1}{\frac{g_j}{N_j} - 1} = -\ln\left(\frac{g_j}{N_j} - 1\right)}\right)$$

$$= k_B \sum_{j=1}^{n} \left(g_j \ln\left(\frac{g_j}{g_j - N_j}\right) + N_j \ln\left(1 + e^{-\frac{\alpha}{k_B} - \frac{\beta}{k_B} E_j} - 1\right)\right)$$

$$= k_B \sum_{j=1}^{n} \left(g_j \ln \frac{g_j}{g_j - N_j} - \underbrace{N_j}_{N} \frac{\alpha}{k_B} - \frac{\beta}{k_B} \underbrace{N_j E_j}_{U}\right.$$

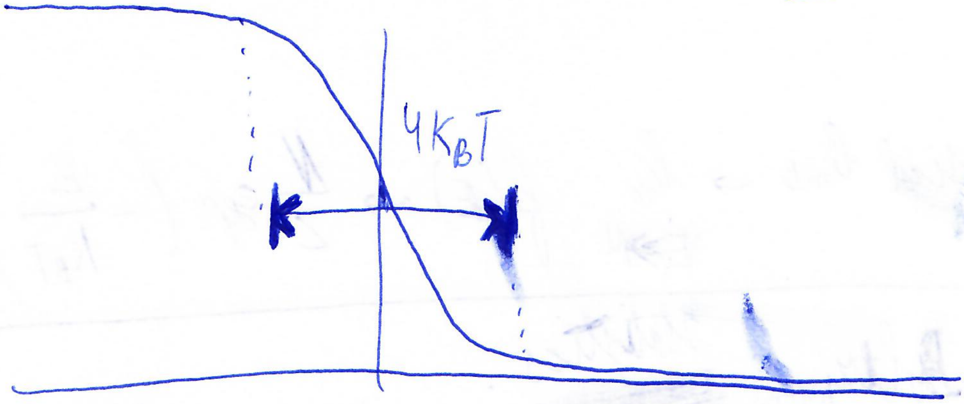
$$\left(\frac{\partial S}{\partial U}\right)_{N,V} = \frac{1}{T} \qquad \left(\frac{\partial S}{\partial N}\right)_{U,V} = -\frac{\mu}{T}$$

$$\beta = -\frac{1}{T} \qquad \alpha = +\frac{\mu}{T}$$

$$\frac{g_j}{N_j} = \exp\left(-\frac{\mu - E_j}{k_B T}\right) + 1$$

$$\left.\begin{aligned} f_j &= \frac{N_j}{g_j} = \left(\exp\left(-\frac{\mu - E_j}{k_B T}\right) + 1\right)^{-1} \\ f(E) &= \frac{N(E)}{g(E)} = \left(\exp\left(-\frac{\mu - E}{k_B T}\right) + 1\right)^{-1} \end{aligned}\right\}$$

Fermi-Dirac distribution



Fermi energy $\lim_{T\to 0} f(E) = H(\mu - E) \Longrightarrow E_F = \lim_{T\to 0} \mu(T)$ | Fermi temperature $T_F = \frac{E_F}{K_B}$

$$N = \int_0^{\infty} N(E)\,dE = \int_0^{\infty} g(E)\,f(E)\,dE$$

$$\lim_{T\to 0} N = \int_0^{\mu(0)} g(E)\,dE$$

$$E_F = \mu(0) = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{\frac{2}{3}}$$

$$T_F = \frac{h^2}{2mk_B}\left(\frac{3N}{8\pi V}\right)^{\frac{2}{3}}$$

Bosons

- indistinguishable
- integer spin
- $\underline{\underline{\underline{\text{no}}}}$ Pauli exclusion
- Bose - Einstein statistics

Classical limit $\rightarrow \lim_{E \gg \mu} f(E) \rightarrow \frac{N}{Z} \exp\left(-\frac{E}{k_BT}\right)$

Maxwell-Boltzmann statistics :

$$W_{MB} = \frac{1}{N!} W_B = \prod_{j=1}^{n} \frac{g_j^{N_j}}{N_j!}$$

$$đW = -\sum_{j=1}^{n} N_j \, dE_j \qquad đQ = \sum_{j=1}^{n} E_j \, dN_j$$

$$\ln\left(\left(\frac{V_f}{V_i}\right)^{-\frac{R}{C_V}}\right) = \ln\left(\frac{T_f}{T_i}\right)$$

$$\left(\frac{V_i}{V_f}\right)^{\frac{R}{C_V}} = \frac{T_f}{T_i}$$

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\frac{R}{C_V}}$$

f → A

i → C

$$T_C = T_A\left(\frac{V_C}{V_A}\right)^{\frac{R}{C_V}}$$

$$T_A = \frac{T_C}{\left(\frac{V_C}{V_A}\right)^{\frac{R}{C_V}}}$$

$$T_A = T_C\left(\frac{V_A}{V_C}\right)^{\frac{R}{C_V}}$$

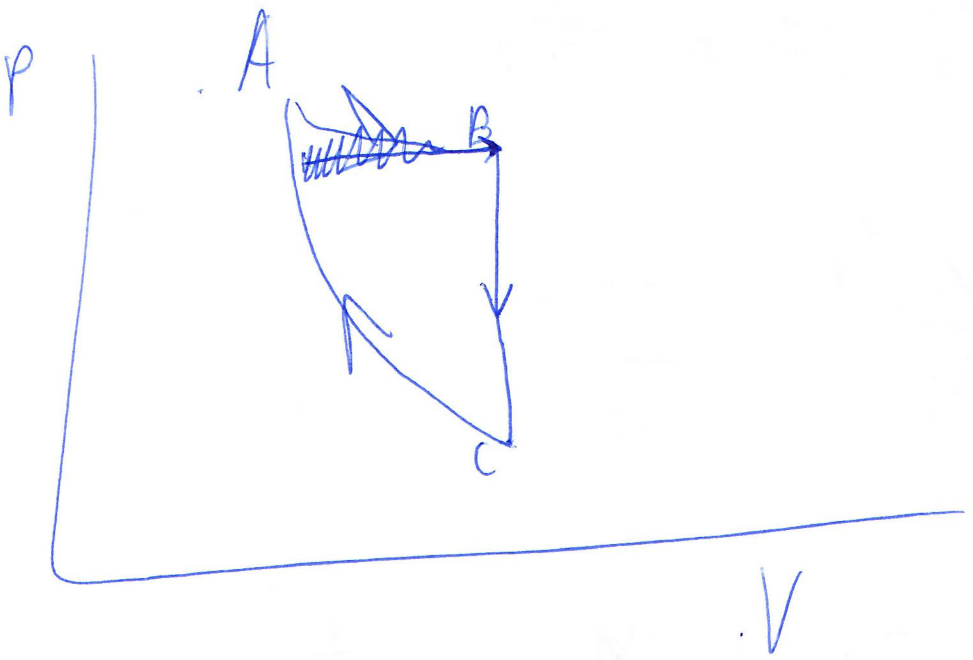
$V_C = V_B = 2V_A$

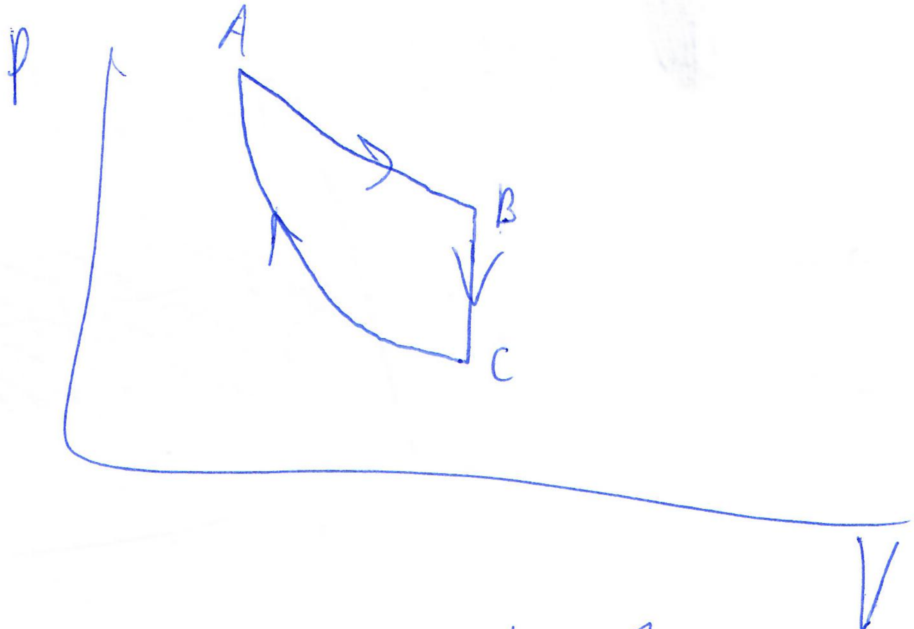
$T_A = 500$ **K**

fill in!

10

$PV = nRT$





$dU = dQ - dW$

adiabatic: $dQ = 0$

$dU = -dW = -PdV$

$n\,C_v\,dT = -\frac{nRT}{V}\,dV$

$\frac{C_v}{T}\,dT = -\frac{R}{V}\,dV$

$C_v \ln\left(\frac{T_f}{T_i}\right) = -R \ln\left(\frac{V_f}{V_i}\right)$

$$dU = \underset{\underset{\text{1 mole}}{\uparrow}}{n} C_v dT = C_v dT$$

$$W = P dV$$

d's should be Δ's down here

step 1: $dU = C_v dT = 0$ (isothermal)

$$dW = P dV = \frac{\overset{\text{1 mole}}{\cancel{n}} R T}{V} dV = RT\ln\left(\frac{V_f}{V_i}\right) = RT\ln 2 = 500 R\ln 2$$

$$dU = dQ - dW \Rightarrow dQ = dW$$
$$= RT\ln 2$$
$$= 500 R\ln 2$$

step 2:

$$dU = C_v dT = C_v(T_C - T_A) = \overset{\overset{(3/2)R}{\downarrow}}{C_v}(T_c - 500) = ?$$

$$dQ = C_v(T_c - 500) \Longleftarrow dU = dQ$$

$$dW = P dV = P \cdot 0 = 0 \text{ (isochoric)}$$

step 3: $dQ = 0$ (adiabatic)

$$dU = -dW$$

$$dU = C_v dT = C_v(T_A - T_C) = C_v(500 - T_c)$$

$$dW = -C_v(500 - T_c)$$

e) net (total (sum of) work should be positive

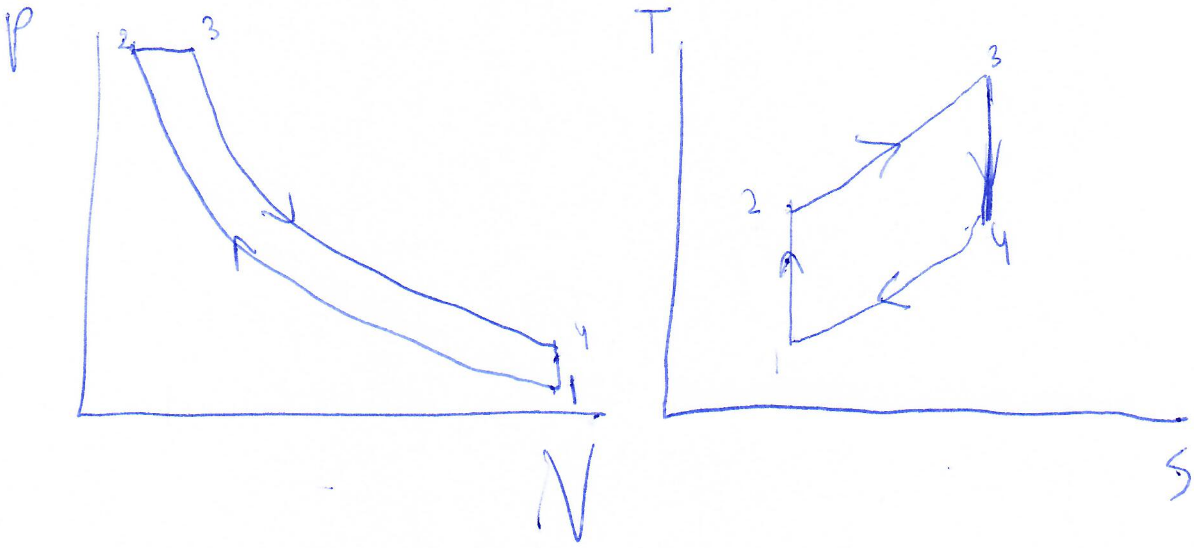
f) $\eta = \frac{|W_{tot}|}{|Q_{in}|} = \frac{\Sigma W}{\sum_{\text{positive } Q} Q}$

adiabatic : $dq = 0$ $ds = 0$

$$PV^{\gamma} = PV^{\frac{C_p}{C_v}} = \text{constant}$$

isothermal : $pV = \text{constant}$

$dT = 0$



$$dU = d\overset{\swarrow = 0,\ \text{adiabatic}}{Q} - dW$$

$$nC_v dT = - dW$$

$$dW = -nC_v dT$$

$$W = -nC_v(T_f - T_i)$$

$$f(x) = \frac{e^{-x/2}}{1 - e^{-x}} \longrightarrow f'(x) = \frac{(1-e^{-x})\, e^{-x/2} \cdot -\frac{1}{2} - e^{-x/2} \cdot e^{-x}}{(1-e^{-x})^2}$$

$$\downarrow$$

$$= \frac{-\frac{1}{2} e^{-\frac{x}{2}}}{(1-e^{-x})^2} - \frac{e^{-x/2 - x}}{(1-e^{-x})^2}$$

$$f'(0) = \frac{-\frac{1}{2}}{1^2} - \frac{1}{1^2} = -\frac{3}{2}$$

statistical postulate : states with equal energy have equal probability

ergodic hypothesis : ensemble averages are equal to time averages

fermions : particles with half-integer spin ; obey Pauli exclusion principle

bosons : particles with (whole-)integer spin ; do not obey the Pauli exclusion principle

it is possible to dispose the exponential in the partition function by changing it to the form $\exp(-\underset{\uparrow \atop \text{constant}}{\alpha} \ \underset{\uparrow \atop \text{quantum number}}{x})$ and then using the Gaussian integral

$$Z = \sum_{s=0}^{\infty} e^{-\frac{\hbar \omega s}{k_B T}} \ \frac{-E_0}{k_B T}$$

$$= \exp(-E_0/k_B T) \sum_{s=0}^{\infty} e^{-\frac{\theta s}{T}}$$

$$\downarrow$$

$$\int_0^{\infty} e^{-\frac{\theta}{T} s} \, ds = \left[-\frac{\theta}{T} e^{-\frac{\theta}{T} s} \right]_0^{\infty}$$

= [illegible] $- -\frac{\theta}{T} e^{-\frac{\theta}{T}}$

$$= \frac{\theta}{T} e^{-\frac{\theta}{T}}$$

$$\mu = \left(\frac{\partial F}{\partial N}\right)_{\text{[illegible]},T}$$

$$\mu = \left(\frac{\partial F}{\partial N}\right)_{V,T}$$

$$\frac{\partial \ln\left(Z_E^{\,N}\right)}{\partial N} = \frac{\partial N \ln\left(Z_E\right)}{\partial N} = z_E$$

$$F = -k_B T \ln Q$$
$$= -k_B T \ln\left(Z_E^N\right)$$

$$\mu_E = \left(\frac{\partial F}{\partial N}\right)_{T,V} = -k_B T \cdot \frac{1}{Z_E^N} \frac{\partial Z_E^{\,N}}{\partial N}$$

$$= -k_B T \frac{1}{Z_E^N} Z_E^N \ln\left(Z_E\right)$$

$$= -k_B T \ln\left(Z_E\right) \quad \square$$

$$\frac{1}{a} \frac{\partial b}{\partial\left(\frac{1}{a}\right)} = -a \frac{\partial b}{\partial a}$$

$$\sum E \frac{1}{Z} e^{-\frac{E}{k_B T}} = \frac{1}{Z} \sum \left(\frac{\partial}{\partial -1/(k_B T)}\right) e^{-\frac{E}{k_B T}} = \frac{1}{Z} \frac{\partial}{\partial\left(\frac{-1}{k_B T}\right)} \underbrace{\sum e^{-\frac{E}{k_B T}}}_{Z}$$

$$= -\frac{1}{Z} \frac{\partial Z}{\partial\left(\frac{1}{k_B T}\right)} = -\left(\frac{\partial \ln Z}{\partial\left(\frac{1}{k_B T}\right)}\right)$$

$n = \frac{P_0 V_0}{R T_0}$

ideal gas: $\frac{P_A V_A}{T_A} = \frac{P_B V_B}{T_B} = \frac{P_C V_C}{T_C} \cdots$

$P_C = P_B = P_0 \cdot 3^{\frac{5}{3}}$

$V_C = V_0$

$$T_C = \frac{P_0 \cdot 3^{\frac{5}{3}} \cdot V_0}{\frac{P_0 V_0}{R T_0} \cdot R} = 3^{\frac{5}{3}} \cdot T_0 = 1872\,K$$

g) $\eta = \frac{|W|}{|Q_{in}|}$ ~~$= \frac{|Q_{in}| - |Q_{out}|}{|Q_{in}|}$~~

sum of all work (posi- and negative)

sum of positive heats

$= \frac{6.34}{25.95}$

$dU = TdS - PdV$

$= dQ - dW$

$dW = PdV$

$dU = n c_v dT = C_v dT \rightarrow \Delta U = C_v (T_f - T_i)$

$H = U + PV$

$dH = TdS - PdV + PdV + VdP$

$= TdS + VdP$

$dH = n c_p dT = C_p dT$

$\Delta H = C_p (T_f - T_i)$

$dU = TdS - PdV$

$dS = \frac{dU + PdV}{T}$

$\frac{C_V}{T} dT$

$\frac{nR}{V} dV$

$n = 1$

$\frac{PV}{nR}$

$$dU = C_v\, dT = -dW = -P\,dV = -\frac{nRT}{V}\,dV$$

$$\frac{C_v}{T}\,dT = -\frac{nR}{V}\,dV$$

$$C_v \ln\left(\frac{T_f}{T_i}\right) = -nR \ln\left(\frac{V_f}{V_i}\right)$$

$$\ln\left(\frac{T_f}{T_i}\right) = -\frac{nR}{C_v} \ln\left(\frac{V_f}{V_i}\right)$$

$$\frac{T_f}{T_i} = \left(\frac{V_f}{V_i}\right)^{-\frac{nR}{C_v}} \rightarrow = -\frac{R}{\underset{\overset{\uparrow}{\text{small } c_v}}{c_v}}$$

$$T_f = T_i \left(\frac{V_f}{V_i}\right)^{-\frac{1}{c}}$$

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\frac{1}{c}}$$

$$T_B = T_A \left(\frac{V_A}{V_B}\right)^{\frac{1}{c}} = 300 \left(\frac{V_0}{\frac{1}{3}V_0}\right)^{\frac{R}{\frac{3}{2}R}} = 300 \cdot 3^{\frac{2}{3}}$$

$$p_B = \frac{\overset{\overset{\frac{p_0V_0}{RT_0}}{\downarrow}}{n}RT_B}{\underset{\underset{\frac{1}{3}V_0}{\uparrow}}{V_B}} = 3p_0\frac{T_B}{T_0} = p_0 \cdot 3 \cdot \frac{300 \cdot 3^{\frac{2}{3}}}{300} = p_0 \cdot 3^{\frac{5}{3}}$$

↓ [illegible]

$$+\frac{2RT}{(v-b)^3} \quad \frac{4a}{T(v+c)^3(v-b)} = 0 = \frac{2RT}{(v-b)^3} - \frac{6a}{T(v+c)^4}$$

$$\frac{4a}{T(v+c)^3(v-b)} = \frac{6a}{T(v+c)^4}$$

$$\frac{4}{(v-b)} = \frac{6}{(v+c)}$$

$$4v + 4c = 6v - 6b$$

$$4c + 6b = 2v$$

$$v = 3b + 2c$$

$$p = \frac{RT}{v-b} - \frac{a}{T(v+c)^2}$$

$$\frac{-2RT}{(v-b)^3} + \frac{4a}{T(v+c)^3(v-b)} = 0$$

$$\left(\frac{\partial p}{\partial v}\right)_T = \frac{-RT}{(v-b)^2} + \frac{aT(2v+2c)}{T^2(v+c)^4}$$

$$= \frac{-RT}{(v-b)^2} + \frac{2a}{T(v+c)^3} = 0$$

$$\left(\frac{\partial^2 p}{\partial v^2}\right)_T = \frac{RT(2v-2b)}{(v-b)^4} - \frac{2aT\,3(v+c)^2}{T^2(v+c)^6}$$

$$= \frac{2RT}{(v-b)^3} - \frac{6a}{T(v+c)^4} = 0$$

$$\frac{2RT}{(v-b)^3} + \frac{RT}{(v-b)^2} = \frac{2a}{T(v+c)^3} + \frac{6a}{T(v+c)^4}$$

$$\frac{RT}{(v-b)^2}\left(\frac{2}{v-b} + 1\right) = \frac{2a}{T(v+c)^3}\left(1 + \frac{3}{(v+c)}\right)$$

$$\left(\frac{\partial v}{\partial T}\right)_p = 3aT^3 \qquad \left(\frac{\partial v}{\partial p}\right)_T = -b$$

$$v = \frac{3}{4}aT^4 + C \qquad v = -bp + C$$

$$-bp = \frac{3}{4}aT^4$$

$$p = -\frac{3}{4}aT^4$$

~~$p = p(v,T) = \left(\frac{\partial p}{\partial v}\right)_T dv$~~

$$v = v(p,T) = \left(\frac{\partial v}{\partial p}\right)_T dp + \left(\frac{\partial v}{\partial T}\right)_p dT$$

$$= -b\,dp + 3aT^3\,dT + C'$$

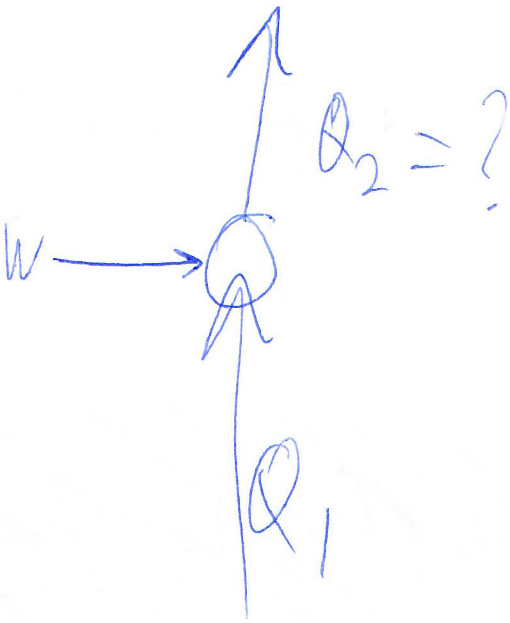
$$= -bp + \frac{3}{4}aT^4 + C'$$

$$bp = \frac{3}{4}aT^4 - v + C'$$

$$p = \frac{3aT^4}{4b} - \frac{v}{b} + C$$

$$CP = \frac{Q \text{ supply by pump}}{E \text{ used}} = \frac{Q_2}{W}$$

$$= \frac{Q_2}{Q_2 - Q_1} = \frac{T_2}{T_2 - T_1}$$



U $\quad dU = Tds - pdV$

H $\quad H = U + PV$

$$dH = TdS - pdV + pdV + VdP$$
$$= TdS + VdP$$

F

$F = U - TS$ $\quad dF = Tds - pdV\ \cancel{M} - Tds - SdT$

$$= -pdV - sdT$$

G

$G = U - TS + Vp$ $\quad dG\ \cancel{= Tds - pdV - Tds} - SdT + VdP\ \cancel{+ pdV}$

$$= VdP - SdT$$

$$\left(\frac{\partial U}{\partial S}\right)_V = T \qquad \left(\frac{\partial U}{\partial V}\right)_S = -p$$

$$\left(\frac{\partial H}{\partial S}\right)_p = T \qquad \left(\frac{\partial H}{\partial p}\right)_S = V$$

$$\left(\frac{\partial F}{\partial V}\right)_T = -p \qquad \left(\frac{\partial F}{\partial T}\right)_V = -S$$

$$\left(\frac{\partial G}{\partial p}\right)_T = V \qquad \left(\frac{\partial G}{\partial T}\right)_p = -S$$

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V$$

$$\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p$$

$$-\left(\frac{\partial p}{\partial T}\right)_V = -\left(\frac{\partial S}{\partial V}\right)$$

$$\left(\frac{\partial V}{\partial T}\right)_p = -\left(\frac{\partial S}{\partial p}\right)_T$$

$$dU = TdS - PdV$$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - p = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

$$= \frac{nRT}{V-nb} - \frac{nRT}{V-nb} + \frac{an^2}{V^2} = \frac{an^2}{V^2}$$

$$dF = dU - TdS - SdT = -PdV - SdT$$

since dF is an exact differential : $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$

$$dH = \underbrace{TdS}_{=0} + VdP$$

~~$\left(\frac{\partial H}{\partial T}\right)_S = V\left(\frac{\partial p}{\partial T}\right)_S = V\left(\frac{\partial S}{\partial V}\right)_p$~~

$$\frac{1}{\left(\frac{\partial S}{\partial p}\right)_T} = \frac{1}{-\left(\frac{\partial V}{\partial T}\right)_p} = -\frac{1}{\beta V}$$

$$\left(\frac{\partial H}{\partial T}\right)_S = V\left(\frac{\partial p}{\partial T}\right)_S$$

$$\left(\frac{\partial p}{\partial T}\right)_S \left(\frac{\partial T}{\partial S}\right)_p \left(\frac{\partial S}{\partial p}\right)_T = -1 \Rightarrow \left(\frac{\partial p}{\partial T}\right)_S = -\overbrace{\left(\frac{\partial S}{\partial T}\right)_p}^{\frac{C_p}{T}} \left(\frac{\partial p}{\partial S}\right)_T$$

Conclusion: $\left(\frac{\partial H}{\partial T}\right)_S = \frac{VC_p}{T\beta V} = \frac{C_p}{\beta T}$

$$\left(\frac{\partial H}{\partial T}\right)_S = \frac{C_p}{\beta T}$$

$$dH = \frac{C_p}{\beta T}\,dT$$

$$= \frac{C_p}{\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p T}\,dT = \frac{C_p}{\frac{1}{V}\frac{nR}{p}T}\,dT = \frac{C_p}{\left(\frac{nRT}{pV}\right)}\,dT$$

$$\uparrow$$

$$\left(\frac{\partial\left(\frac{nRT}{p}\right)}{\partial T}\right)_p$$

$$\overset{\text{idealgaslaw}}{\overset{\downarrow}{=}} \frac{C_p}{\left(\frac{pV}{pV}\right)}\,dT = C_p\,dT = C_p(T_2 - T_1)$$

$$= n\,c_p(T_2 - T_1)$$

$$dU = T\,dS - p\,dV$$

$$dS = \frac{dU}{T} + \frac{p}{T}dV = \frac{dU}{T} + \frac{nR}{V}dV$$

$$= \frac{nC_v}{T}dT + \frac{nR}{V}dV$$

$$\Delta S = 0 + nR\ln\left(\frac{V_f}{V_i}\right)$$

$$= \cancel{nR\ln\left(\frac{V_i}{V_f}\right)} + nR\ln\left(\frac{p_i}{p_f}\right)$$

$$pV = nRT$$

$$V = \frac{nRT}{p}$$

$$p = \frac{nRT}{V-nb} - a\frac{n^2}{V^2}$$

$$\left(\frac{\partial H}{\partial T}\right)_S \left(\frac{\partial T}{\partial S}\right)_H \left(\frac{\partial S}{\partial H}\right)_T = -1$$

$$\left(\frac{\partial H}{\partial T}\right)_S = -\left(\frac{\partial S}{\partial T}\right)_H \left(\frac{\partial H}{\partial S}\right)_T$$

$$dH = \overset{dQ_r = 0}{\overset{\downarrow}{T\,dS}} + V\,dP$$

$$= V\,dP$$

$$\left(\frac{\partial H}{\partial p}\right) = V$$

$$dH = T\,dS + V\,dp = V\,dp$$

$$\left(\frac{\partial H}{\partial T}\right)_S =$$

$$= \frac{\left(\frac{\partial H}{\partial T}\right)_p}{\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p T} = \frac{C_p}{\beta T}$$

U

$dU = T\,ds - p\,dV$

$H = U + PV$

$dH = dU + d(PV)$

$= T\,dS - P\,dV + P\,dV + V\,dP$

$= T\,dS + V\,dP$

$F = U - TS$

$dF = dU - d(TS)$

$= T\,dS - P\,dV - T\,dS - S\,dT$

$= -P\,dV - S\,dT$

$G = U - TS + VP$

$dG = dU - d(TS) + d(VP)$

$= T\,dS - P\,dV - T\,dS - S\,dT + P\,dV + V\,dP$

$= V\,dP - S\,dT$

$$\left(\frac{\partial V}{\partial S}\right)_P = \left(\frac{\partial T}{\partial P}\right)_S$$

$\nwarrow$?

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

3C

Using the equation derived in b we get

$$\left(\frac{\partial S}{\partial T}\right)_P = \left(\frac{\partial S}{\partial T}\right)_V + \left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P$$

$$\frac{C_p}{T} - \frac{C_v}{T} = \left(\frac{\partial S}{\partial V}\right)_T \beta V$$

From $F = U - TS$ and $dF = dU - d(Ts) = TdS - PdV - TdS - SdT$
$= -PdV - SdT$ follows the

maxwell relation $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

Cyclical rule: $\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_P \left(\frac{\partial V}{\partial P}\right)_T = -1$

$$\left(\frac{\partial P}{\partial T}\right)_V = - \frac{\left(\frac{\partial V}{\partial T}\right)_P}{\left(\frac{\partial V}{\partial P}\right)_T} = - \frac{\beta V}{-\kappa V} = \frac{\beta}{\kappa}$$

Substituting in the earlier equation gives $C_p - C_v = \frac{\beta^2 T}{\kappa} V$

3b

$$\left(\frac{\partial S}{\partial T}\right)_p = ?$$

Using the 'change of variable' rule from the formula sheet (which is allowed, since S is a state variable and thus depends only on P, V and T):

with $f \to S$

$x \to T$

$y \to V$

$z \to P$

$$\left(\frac{\partial S}{\partial T}\right)_p - \left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p$$

Re-arranging gives us

$$\left(\frac{\partial S}{\partial T}\right)_p = \left(\frac{\partial S}{\partial T}\right)_V + \left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p \qquad \square$$

3a

$$dS = \left(\frac{\partial S}{\partial V}\right)_T dV + \underbrace{\left(\frac{\partial S}{\partial T}\right)_V}_{\to \frac{C_v}{T}} dT$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V \to \frac{nR}{V-nb}$$

$$p = \frac{nRT}{V-nb} - a\frac{n^2}{V^2}$$

~~$dU = TdS - pdV$~~

~~$dS = \frac{dU}{T} + \frac{p}{T} dV$~~

~~$dS = 0$, (reversible adiabatic)~~

~~$\frac{dU}{T} = -\frac{p}{T} dV$~~

~~$\frac{dU}{T} = -\left(\frac{nR}{V-nb} - a\frac{n^2}{V^2}\right) dV$~~

$$0 = \frac{C_v}{T} dT + \frac{nR}{V-nb} dV$$

$$C_v \ln\left(\frac{T_f}{T_i}\right) = -nR \ln\left(\frac{V_f - nb}{V_i - nb}\right)$$

$dU = TdS - PdV$ $= 0$, ideal gas $dT = 0$

nRT

$$dS = \frac{dU}{T} - \frac{P}{T}\,dV$$ ~~$= -nR\ln\left(\frac{V_f}{V_i}\right) = -nR\ln\left(\frac{\frac{nRT}{P_f}}{\frac{nRT}{P_i}}\right)$~~

~~$= -nR\ln\left(\frac{P_i}{P_f}\right)$~~

$$dS = \frac{dQ_r}{T}$$

$H = U + PV$

$$dH = dU + d(PV) = TdS - PdV + PdV + VdP$$

$$= TdS + \overset{\frac{nRT}{P}}{V}dP$$

$$dS = \frac{dH}{T} - \frac{V}{T}\,dP$$

$$= \frac{dH}{T} - nR\ln\left(\frac{P_f}{P_i}\right)$$

$PV = nRT$

		dQ	dW
$1 \rightarrow 2$	adiabat	0	$-n C_v(T_f - T_i)$
$2 \rightarrow 3$	isobar	$n C_p (T_3 - T_2)$	$P dV = \frac{nRT}{V} dV = nRT \ln\left(\frac{V_f}{V_i}\right.$
$3 \rightarrow 4$	adiabat	0	$-n C_v (T_f - T_i)$
$4 \rightarrow 1$	isobar	$n C_p (T_1 - T_4)$	$P dV = \frac{nRT}{V} dV = nRT \ln\left(\frac{V_f}{V_i}\right.$

$n c_v dT = dU = dQ - dW$

$dQ = n c_p dT$ (isobar) (!) ! ! ! !

$\Delta Q = n c_p (T_f - T_i)$

$$\eta = \frac{|W|}{|Q_{in}|} = \frac{|Q_{in}| - |Q_{out}|}{|Q_{in}|} = 1 - \frac{|Q_{out}|}{|Q_{in}|}$$

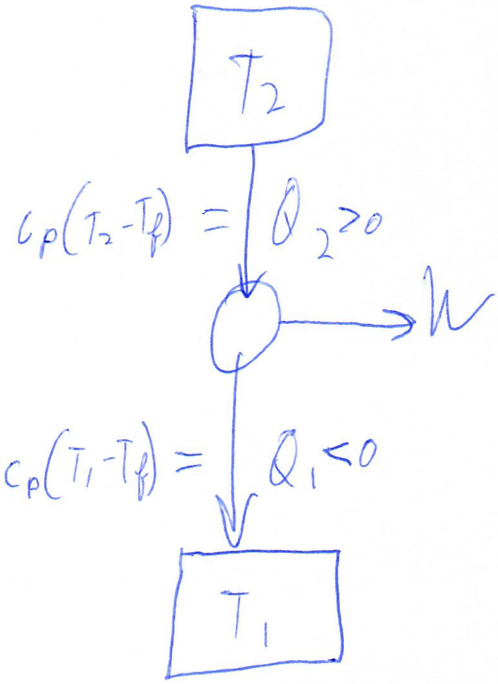
$=$ [illegible]

$$1 - \frac{|C_p(T_1 - T_4)|}{|C_p(T_3 - T_2)|} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

Einstein crystal, 3D-oscillator, N copies

$$f\left(\frac{\theta_E}{T}\right) \frac{e^{-\frac{\theta_E}{2T}}}{1-e^{-\frac{\theta_E}{T}}} \approx \frac{1+\overset{\text{not needed}}{\left(-\frac{\theta_E}{2T}\right)}+\overset{\text{not needed}}{\left(-\frac{\theta_E}{2T}\right)^2}\ldots}{1-1-\underset{\text{needed, because terms 1 cancel out}}{\left(-\frac{\theta_E}{T}\right)}\ldots} \approx$$

$$\frac{1-\frac{\theta_E}{2T}}{\frac{\theta_E}{T}} \approx \frac{T}{\theta_E}$$



$$= Q_2 + Q_1 = c_p\left(T_2+T_1-2T_f\right)$$

$$P = \frac{nRT}{V-nb} - a\frac{n^2}{V^2}$$

$$dU = TdS - PdV$$

$$\left(\frac{\partial U}{\partial V}\right)_T = ?$$

$$\left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_U \left(\frac{\partial T}{\partial U}\right)_V = -1$$

$$\left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_U \frac{1}{C_V} = -1$$

$$\left(\frac{\partial U}{\partial V}\right)_T = -C_V \left(\frac{\partial T}{\partial V}\right)_U$$

$$\frac{1}{= \left(\frac{\partial U}{\partial T}\right)_V} = \frac{1}{\underset{\substack{\uparrow \\ \text{capital}}}{C_V}} \neq 0$$

~~$-TdS\ dU$~~

~~$dU = TdS - PdV$~~

e) $dU =$

$$\left(\frac{\partial T}{\partial V}\right)_U \left(\frac{\partial V}{\partial U}\right)_T \overset{\overset{?}{=}0}{\left(\frac{\partial U}{\partial T}\right)_V} = -1 \Rightarrow \left(\frac{\partial T}{\partial V}\right)_U = 0 = -\underset{\substack{\uparrow \\ \text{or}}}{\left(\frac{\partial U}{\partial V}\right)_T} \underset{\substack{\nearrow \\ =0}}{\left(\frac{\partial T}{\partial U}\right)_V} = 0$$

$=0$

$$u = \int_0^\infty N(E)\,E\,dE$$

$$E_F = \mu(T \overset{?}{=} 0)$$

$$T_F = E_F / k_B$$

$$dU = T\,dS - P\,dV$$

$$dS = \boxed{\frac{dU}{T}} + \frac{P}{T}\,dV$$

$$\frac{1}{T}\,dU = \frac{1}{T}\frac{dU}{dT}\,dT$$

$$= \frac{1}{T}\,C_V\,dT$$

$$dS = \frac{C_V}{T}\,dT + \frac{nR}{V}\,dV$$

$$S = C_V \ln\frac{T_f}{T_i} + nR\ln\frac{V_f}{V_i}$$

$$\frac{PV}{T} = nR$$

$$\frac{P_1V_1}{T_1} = \frac{P_1V_2}{T_2} = \frac{P_2V_2}{T_3}$$

$$\frac{T_2}{T_1} = \frac{V_2}{V_1} \quad \Bigg| \quad \frac{T_3}{T_2} = \frac{P_2}{P_1}$$

$$\Delta S_A = C_V \ln\left(\frac{V_2}{V_1}\right) + nR\ln\left(\frac{V_2}{V_1}\right)$$

$$= C_p \ln\left(\frac{V_2}{V_1}\right)$$

$$\Delta S_B = C_V \ln\left(\frac{T_3}{T_2}\right) + nR\ln 1$$

$$= C_V \ln\left(\frac{P_2}{P_1}\right)$$

$x = 0$

$\Delta S_C = 0$ because

adiabatic process

variables

U

$dU = T\,dS - P\,dV$ $\quad$ S, V

$$\left(\frac{\partial u}{\partial s}\right)_V = T \quad \left(\frac{\partial u}{\partial V}\right)_S = -P$$

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$H = U + PV$ $\quad$ S, P

$dH = dU + d(PV) = T\,dS - P\,dV + P\,dV + V\,dP$

$= T\,dS + V\,dP$

$$\left(\frac{\partial H}{\partial s}\right)_P = T \quad \left(\frac{\partial H}{\partial P}\right)_S = V$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$F = U - TS$ $\quad$ T, V

$F = dU - d(TS) = T\,dS - P\,dV - S\,dT - T\,dS$

$= -P\,dV - S\,dT$

$$\left(\frac{\partial F}{\partial V}\right)_T = -P \quad \left(\frac{\partial F}{\partial T}\right)_V = -S$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

$G = U - TS + VP$ $\quad$ P, T

$G = dU - d(TS) + d(VP)$

$= T\,dS - P\,dV - T\,dS - S\,dT + V\,dP + P\,dV$

$= V\,dP - S\,dT$

$$\left(\frac{\partial G}{\partial P}\right)_T = V \quad \left(\frac{\partial G}{\partial T}\right)_P = -S$$

$$\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$$