Exercise 2-6

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$$p = \frac{RT}{v-b} - \frac{a}{Tv^2}$$

$$\frac{\partial p}{\partial v} = 0 \rightarrow \frac{-RT}{(v-b)^2} - \frac{-a2Tv}{T^2v^4} = 0 \rightarrow T\left(\frac{2a}{T^2v^3} - \frac{R}{(v-b)^2}\right) = 0$$

$$\frac{2a}{T^2v^3} = \frac{R}{(v-b)^2} \rightarrow RT^2v^3 = 2a(v-b)^2$$

$$\frac{\partial^2 p}{\partial v^2} = 0 \rightarrow T\left(\frac{-2a\,3T^2v^2}{T^4v^6} - \frac{-R(2v-2b)}{(v-b)^4}\right) = 0$$

$$\frac{-6a}{T^2v^4} = \frac{-2R}{(v-b)^3}$$

$$2RT^2v^4 = 6a(v-b)^3$$

$$2v\left(2a(v-b)^2\right) = 6a(v-b)^3$$

$$4av(v-b)^2 = 6a(v-b)^3$$

$$2v(v-b)^2 = 3(v-b)^3$$

$$2v(v^2 - 2vb + b^2) = 3(v^3 - 2v^2b + vb^2 - bv^2 + 2vb^2 - b^3)$$

$$2v^3 - 4v^2b + 2vb^2 = 3v^3 - 9v^2b + 9vb^2 - 3b^3$$

$$v^3 - 5v^2b + 7vb^2 - 3b^3 = 0$$

$$2v - 3(v-b) = 3v - 3b$$

$$-v = -3b$$

$$v = 3b$$

$$v_c = 3b$$

$$RT^2v^3 = 2a(v-b)^2$$

$$RT^2 27b^3 = 2a(2b)^2$$

$$RT^2 27b^3 = 8ab^2$$

$$27RT^2b = 8a$$

$$T^2 = \frac{8}{27}\frac{a}{Rb}$$

$$T_c = +\sqrt{\frac{8}{27}\frac{a}{Rb}}$$

$$p_c = \frac{RT}{v-b} - \frac{a}{Tv^2} = \frac{R\sqrt{\frac{8}{27}\frac{a}{Rb}}}{3b-b} - \frac{a}{\sqrt{\frac{8}{27}\frac{a}{Rb}}\,3^2b^2} = \frac{\sqrt{\frac{8}{27}\frac{aR}{b}}}{2b} - \sqrt{\frac{27Rba^2}{8a^3b^4}}$$

$$= \sqrt{\frac{2}{27}\frac{aR}{b^3}} - \sqrt{\frac{1}{24}\frac{aR}{b^3}}$$

$$= \left(\sqrt{\frac{2}{27}} - \sqrt{\frac{1}{24}}\right)\sqrt{\frac{aR}{b^3}}$$

$$= \frac{1}{12b}\left(\sqrt{\frac{2\cdot 144}{27}} - \sqrt{\frac{1\cdot 144}{24}}\right)\sqrt{\frac{aR}{b}}$$

$$\frac{1}{12b}\left(\sqrt{\frac{32}{3}} - \sqrt{6}\right)\sqrt{\frac{aR}{b}}$$

$$= \frac{1}{12b}\left(\sqrt{32} - \sqrt{18}\right)\sqrt{\frac{aR}{3b}}$$

$$= \frac{1}{12b}\left(4\sqrt{2} - 3\sqrt{2}\right)\sqrt{\frac{aR}{3b}}$$

$$= \frac{1}{12b}\sqrt{\frac{2aR}{3b}}$$

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