Exercise 2-7

Wednesday, 22 April 2020 15:40

$$\frac{\partial P}{\partial v} = \frac{-RT}{(v-b)^2} e^{-\frac{a}{RTv}} - \frac{RT}{v-b} e^{-\frac{a}{RTv}} \cdot \left(-\frac{aRT}{R^2T^2v^2}\right) = \frac{-RT}{(v-b)^2} e^{-\frac{a}{RTv}} + \frac{a}{v^2(v-b)} e^{-\frac{a}{RTv}} \rightarrow \frac{RT}{v-b} = \frac{a}{v^2} \rightarrow RTv^2 = av - ab$$

$$RT = \frac{a(v-b)}{v^2}$$

$$\frac{\partial^2 P}{\partial v^2} = \frac{RT(2v-2b)}{(v-b)^4} e^{-\frac{a}{RTv}} + \left(-\frac{aRT}{R^2T^2v^2}\right) \cdot \frac{-RT}{(v-b)^2} e^{-\frac{a}{RTv}} + \frac{-a\cdot(3v^2-2bv)}{v^4(v-b)^2} e^{-\frac{a}{RTv}} + \frac{a}{v^2(v-b)} \cdot \frac{-aRT}{R^2T^2v^2} e^{-\frac{a}{RTv}} = 0$$

$$\frac{2RT(v-b)}{(v-b)^4} - \frac{a}{v^2(v-b)^2} - \frac{av(3v-2b)}{v^4(v-b)^2} + \frac{a^2}{v^4(v-b)RT} = 0$$

$$\frac{2a(v-b)}{v^2(v-b)^3} - \frac{a}{v^2(v-b)^2} - \frac{av(3v-2b)}{v^4(v-b)^2} + \frac{a^2v^2}{v^4(v-b)a(v-b)} = 0$$

$$\frac{2a}{v^2} - \frac{a}{v^2} - \frac{a}{v^3}(3v-2b) + \frac{a}{v^2} = 0$$

$$2a - \frac{a}{v}(3v-2b) = 0$$

$$2a = \frac{a}{v}(3v-2b)$$

$$2a = 3a - \frac{2ab}{v}$$

$$a = \frac{2ab}{v}$$

$$v = 2b$$

$$RT = \frac{a(v-b)}{v^2}$$

$$T = \frac{a(2b-b)}{2^2b^2R} = \frac{ab}{4b^3R} = \frac{a}{4bR}$$

$$e^{-\frac{a}{RTv}} = e^{-\frac{a\,4bR}{Ra\,2b}} = e^{-2}$$

$$P = \frac{RT}{v-b} e^{-\frac{a}{RTv}} = \frac{Ra}{4bR(2b-b)} e^{-2} = \frac{a}{4b^2e^2}$$