Exercise 2.9

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$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$$

$$\kappa = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

$$\left(\frac{\partial \beta}{\partial p}\right)_T = \left(\frac{\partial}{\partial p}\frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p\right)_T$$

$$\left(\frac{\partial \kappa}{\partial T}\right)_p = \left(\frac{\partial}{\partial T}\cdot -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T\right)_p$$

f g

$$\left(\frac{\partial \beta}{\partial p}\right)_T + \left(\frac{\partial \kappa}{\partial T}\right)_p = \left(\frac{\partial}{\partial p}\frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p\right)_T - \left(\frac{\partial}{\partial T}\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T\right)_p$$

product rule

chain rule

$$= -\frac{1}{v^2}\left(\frac{\partial v}{\partial p}\right)_T\left(\frac{\partial v}{\partial T}\right)_p + \frac{1}{v}\left(\frac{\partial}{\partial p}\left(\frac{\partial v}{\partial T}\right)_p\right)_T + \frac{1}{v^2}\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial v}{\partial p}\right)_T - \frac{1}{v}\left(\frac{\partial}{\partial T}\left(\frac{\partial v}{\partial p}\right)_T\right)_p$$

df g + f dg

$$= \frac{1}{v}\left(\frac{\partial}{\partial p}\left(\frac{\partial v}{\partial T}\right)_p\right)_T - \frac{1}{v}\left(\frac{\partial}{\partial T}\left(\frac{\partial v}{\partial p}\right)_T\right)_p$$

$$= \frac{1}{v}\left(\left(\frac{\partial}{\partial p}\left(\frac{\partial v}{\partial T}\right)_p\right)_T - \left(\frac{\partial}{\partial T}\left(\frac{\partial v}{\partial p}\right)_T\right)_p\right)$$

Now, because $dv = \left(\frac{\partial v}{\partial T}\right)_p dT + \left(\frac{\partial v}{\partial p}\right)_T dp$ is an exact differential,

$$\left(\frac{\partial}{\partial p}\left(\frac{\partial v}{\partial T}\right)_p\right)_T = \left(\frac{\partial}{\partial T}\left(\frac{\partial v}{\partial p}\right)_T\right)_p$$

$$= \frac{1}{v}\cdot 0$$

$$= 0$$