Exercise 3.11

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ideal gas: $PV = nRT$

a)

$V_i = 1\,m^3 \qquad P_i = 1\,bar = 10^5\,Pa$

$V_f = 1.8\,m^3 \qquad P_f = \frac{1}{2}bar = 0.5 \cdot 10^5\,Pa$

$V = AP + B$

$A = \frac{1.8-1\,m^3}{(0.5-1)\cdot 10^5\,Pa} = \frac{0.8}{-0.5 \cdot 10^5} = -1.6 \cdot 10^{-5}\,m^3\,Pa^{-1}$

$1 = -1.6 \cdot 10^{-5} \cdot 10^5 + B$

$1 = -1.6 + B$

$2.6 = B$

$B = 2.6\,m^3$

b) $T_i = 300\,K$

$P_i V_i = nRT_i$

$P_f V_f = nRT_f$

$P_f = \frac{1}{2}bar = \frac{1}{2} \times 1\,bar = \frac{1}{2} P_i$

$V_f = 1.8\,m^3 = 1.8 \times 1\,m^3 = 1.8\,V_i$

$1.8 \cdot \frac{1}{2} P_i V_i = nRT_f$

$1.8 \cdot \frac{1}{2} nRT_i = nRT_f$

$\frac{T_f}{T_i} = 1.8 \cdot \frac{1}{2} = 0.9$

$T_f = 0.9\,T_i = 0.9 \cdot 300\,K = 270\,K$

c) $dW = PdV$

$V = AP + B \Rightarrow dV = AdP$

$dW = PAdP$

$W = \int_{P_i}^{P_f} PAdP$

$= A\int_{P_i}^{P_f} PdP$

$= \left[\frac{1}{2}AP^2\right]_{P_i}^{P_f}$

$= \frac{1}{2}A\left((\frac{1}{2}10^5)^2 - (10^5)^2\right)$

$= \frac{1}{2} \cdot -1.6 \cdot 10^{-5}\left(-\frac{3}{4} \cdot 10^{10}\right)$

$= 0.6 \cdot 10^5\,J$

d) $dU = dQ - dW$

$U_i = 300 \cdot 800 = 24 \cdot 10^4\,J$

$U_f = 800 \cdot 270 = 21.6 \cdot 10^4\,J$

$dQ = dU + dW$

$= 21.6 \cdot 10^4 - 24 \cdot 10^4 + 6 \cdot 10^4$

$= (-2.4 + 6) \cdot 10^4$

$= 3.6 \cdot 10^4\,J$