Exercise 9.11

$C_V = \frac{5}{2} R$

$V_0 = 2 m^3$

$P_0 = 1 atm = 1.01 \cdot 10^5 Pa$

$T_1 = 20°C = 293 K$

$P_f = 8 atm = 8.08 \cdot 10^5 Pa$

$R = 8.314 \ J mol^{-1} K^{-1}$

$V_f = ?$

$T_f = ?$

$W = ?$

$Q = ?$

$\Delta U = ?$

reversible $\longrightarrow dW = PdV$

isothermal $\Longrightarrow PV =$ constant

adiabatic $\Longrightarrow PV^{\gamma} =$ constant

a) isothermal $\Rightarrow T_f = T_1 = 293 K$

$dU = 0$

$dQ = -dW$

$P_1 V_1 = P_2 V_2$

$P_2 = 2 P_1$

$\Rightarrow P_1 V_1 = 2 P_1 V_2$

$V_1 = 2 V_2$

$V_2 = 1 m^3$

$$W = \int_{V_1}^{V_2} P dV = nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT_1 \ln\left(\frac{V_2}{V_1}\right) = nRT_1 \ln\left(\frac{1}{2}\right) = P_1 V_1 \ln\left(\frac{1}{2}\right) = 5.62 \cdot 10^5 J$$

$\Delta U_{12} = 0$

$Q_{12} = W_{12} = 5.62 \cdot 10^5 J$

b) adiabatic : $dQ = 0$, $PV^{\gamma} =$ constant

$Q = 0$

$dU = -dW$

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

$$V_2^{\gamma} = \frac{P_1}{P_2} V_1^{\gamma}$$

$$V_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} V_1$$

$$\gamma = \frac{C_P}{C_V} = \frac{R + C_V}{C_V} = \frac{R + \frac{5}{2}R}{\frac{5}{2}R} = \frac{7}{5}$$

$$V_2 = \left(\frac{1}{2}\right)^{\frac{5}{7}} \cdot 2 m^3 = 1.22 m^3$$

Ideal gas : $PV = nRT$

$$\frac{PV}{T} = \text{constant}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$T_2 = \frac{P_2 V_2 T_1}{P_1 V_1} = 357 K$$

$dU = C_V dT$

$dU = n C_V dT$

$dW = -dU$

$dW = -n C_V dT$

$= -n C_V (T_2 - T_1)$

$= -\frac{PV}{RT_1} R \frac{5}{2} (T_2 - T_1)$

$= -\frac{5}{2} \frac{4 \cdot 1.01325 \cdot 10^5 \cdot 2}{293} (357 - 293) = -4.44 \times 10^5 J$

$dU = 4.44 \times 10^5 J$