

to show:  $\left(\frac{\partial u}{\partial T}\right)_p = C_p - P \beta v$

$$\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T}\right)_p$$

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p = \left(\frac{\partial(u + P v)}{\partial T}\right)_p$$

$$\begin{aligned} C_p - P \beta v &= \left(\frac{\partial(u + P v)}{\partial T}\right)_p - P \frac{1}{v} \left(\frac{\partial v}{\partial T}\right)_p v \\ &= \left(\frac{\partial u}{\partial T}\right)_p + \left(\frac{\partial(P v)}{\partial T}\right)_p - P \left(\frac{\partial v}{\partial T}\right)_p \end{aligned}$$

$$= \left(\frac{\partial u}{\partial T}\right)_p + P \left(\frac{\partial v}{\partial T}\right)_p - P \left(\frac{\partial v}{\partial T}\right)_p$$

as we can take  $P$  out of the partial derivative, as we have kept it constant

$$= \left(\frac{\partial u}{\partial T}\right)_p$$

