Exercise 4.8

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Variable $\Delta W = P\Delta V$

adiabatic $PV^{\gamma} = K$

ideal gas: $PV = nRT$

$T V^{\gamma-1} = \text{constant}$

$\gamma = \frac{C_p}{C_v}$

$dq = -dw = -P\,dv$

$$dU = \left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT$$

$$= \left(\frac{\partial U}{\partial T}\right)_V dT$$

$$= C_v\, dT$$

$$\Downarrow$$

$$C_v\, dT = dq = -P\,dv$$

$$\Downarrow$$

$$C_v\, dT = -\frac{RT}{v}\, dv$$

$$\Downarrow$$

$$C_v \frac{dT}{T} = -R\,\frac{dv}{v}$$

$$\Downarrow$$

$$C_v \ln\left(\frac{T_f}{T_i}\right) = -R \ln\left(\frac{v_f}{v_i}\right) = R \ln\left(\frac{v_i}{v_f}\right)$$

$$\Downarrow$$

$$\ln\left(\left(\frac{T_f}{T_i}\right)^{C_v}\right) = \ln\left(\left(\frac{v_i}{v_f}\right)^{R}\right)$$

$$\Downarrow$$

$$\left(\frac{T_f}{T_i}\right)^{C_v} = \left(\frac{v_i}{v_f}\right)^{R}$$

$$C_p = C_v + R$$

$$\Downarrow$$

$$R = C_p - C_v$$

$$\Downarrow$$

$$\left(\frac{T_f}{T_i}\right)^{C_v} = \left(\frac{v_i}{v_f}\right)^{C_p - C_v}$$

$$T_f^{\,C_v} T_i^{-C_v} = v_i^{\,C_p - C_v}\, v_f^{\,C_v - C_p}$$

$$\Downarrow$$

$$T_f^{\,C_v} v_f^{\,C_p - C_v} = T_i^{\,C_v} v_i^{\,C_p - C_v}$$

$$\Downarrow$$

$$\left(T_f^{\,C_v} v_f^{\,C_p - C_v}\right)^{\frac{1}{C_v}} = \left(T_i^{\,C_v} v_i^{\,C_p - C_v}\right)^{\frac{1}{C_v}}$$

$$\Downarrow$$

$$T_f\, v_f^{\,\frac{C_p - C_v}{C_v}} = T_i\, v_i^{\,\frac{C_p - C_v}{C_v}}$$

$$\Downarrow$$

$$T_f\, v_f^{\,\gamma - 1} = T_i\, v_i^{\,\gamma - 1}$$

$$\square$$

b) $T V^{\gamma-1} = \text{constant}$

$V = \frac{nRT}{P}$

$$T\left(\frac{nRT}{P}\right)^{\gamma-1} = \text{constant}$$

$$\Downarrow$$

$$T\, P^{-(\gamma-1)}\, T^{\gamma-1} = \frac{\text{constant}}{(nR)^{\gamma-1}}$$

$$\Downarrow$$

$$T^{\gamma}\, P^{1-\gamma} = \text{constant}$$

$$\Downarrow$$

$$\frac{T^{\gamma}}{P^{\gamma-1}} = \text{constant}$$

$$\Downarrow$$

$$\left(\frac{T^{\gamma}}{P^{\gamma-1}}\right)^{\frac{1}{\gamma}} = (\text{constant})^{\frac{1}{\gamma}}$$

$$\Downarrow$$

$$\frac{T}{P^{1-\frac{1}{\gamma}}} = \text{constant}$$

$$\square$$