Exercise 5.2

Thursday, 30 April 2020 19:47

a) $u = u_0 + c_v(T - T_0)$ → $u = u_0 + c_v T - c_v T_0$

↳ $T = \frac{u - u_0}{c_v} + T_0$

↳ Can we say: in partial derivative, u is assumed constant, and all others are assumed constant, thus partial derivative $= 0$?

$\eta = \left(\frac{\partial T}{\partial v}\right)_u$

cyclical relation:

$$\left(\frac{\partial T}{\partial v}\right)_u \left(\frac{\partial v}{\partial u}\right)_T \left(\frac{\partial u}{\partial T}\right)_v = -1$$

$$\eta = \frac{-1}{\left(\frac{\partial v}{\partial u}\right)_T \left(\frac{\partial u}{\partial T}\right)_v} = -\frac{1}{\left(\frac{\partial v}{\partial u}\right)_T c_v} = -\frac{1}{c_v}\left(\frac{\partial u}{\partial v}\right)_T = 0$$

$= 0$

b) Joule-Thompson coefficient $\mu = \left(\frac{\partial T}{\partial P}\right)_h$

$h = u + Pv$

$$\left(\frac{\partial T}{\partial p}\right)_h \left(\frac{\partial p}{\partial h}\right)_T \left(\frac{\partial h}{\partial T}\right)_p = -1$$

$$\left(\frac{\partial T}{\partial p}\right)_h = \frac{-1}{\left(\frac{\partial p}{\partial h}\right)_T \left(\frac{\partial h}{\partial T}\right)_p} = \frac{-\left(\frac{\partial h}{\partial p}\right)_T}{c_p}$$

So, $\left(\frac{\partial T}{\partial p}\right)_h = 0$ if and only if $\left(\frac{\partial h}{\partial p}\right)_T = 0$

As $h = u + Pv$,

$dh = du + P\,dv$

$dh = du + R\,dT$, as $P\,dv = R\,dT$ from the ideal gas law

$dh - du = R\,dT$

Writing the exact differentials for $u = u(v, T)$ and $h = h(T, P)$, we get

$$du = \left(\frac{\partial u}{\partial v}\right)_T dv + \left(\frac{\partial u}{\partial T}\right)_v dT = \left(\frac{\partial u}{\partial T}\right)_v dT = c_v\,dT, \text{ as } \left(\frac{\partial u}{\partial v}\right)_T = 0 \text{ and } \left(\frac{\partial u}{\partial T}\right)_v = c_v$$

$$dh = \left(\frac{\partial h}{\partial P}\right)_T dP + \left(\frac{\partial h}{\partial T}\right)_p dT = \left(\frac{\partial h}{\partial P}\right)_T dP + c_p\,dT, \text{ as } \left(\frac{\partial h}{\partial T}\right)_p = c_p$$

Filling this all in, we get

$$dh - du = c_p\,dT + \left(\frac{\partial h}{\partial P}\right)_T dP - c_v\,dT = R\,dT + \left(\frac{\partial h}{\partial P}\right)_T dP = R\,dT$$

from what was given above and $c_p - c_v = R$, so

$$R\,dT + \left(\frac{\partial h}{\partial P}\right)_T dP = R\,dT,$$

which implies that

$$\left(\frac{\partial h}{\partial P}\right)_T dP = 0$$

leading to

$$\left(\frac{\partial h}{\partial P}\right)_T = 0$$