Exercise 5.3

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$u = u_0 + c_v T - \frac{a}{v}$ $\quad u_0, a, c_v$ constants

a)

$$\eta = \left(\frac{\partial T}{\partial v}\right)_u \rightarrow \left(\frac{\partial T}{\partial v}\right)_u \left(\frac{\partial v}{\partial u}\right)_T \left(\frac{\partial u}{\partial T}\right)_v = -1$$

$$\eta = -\frac{\left(\frac{\partial u}{\partial v}\right)_T}{\left(\frac{\partial u}{\partial T}\right)_v}$$

$$\left(\frac{\partial u}{\partial v}\right)_T = \frac{a}{v^2}$$

$$\left(\frac{\partial u}{\partial T}\right)_v = c_v$$

$$\eta = \frac{-a}{c_v v^2}$$

Van der Waals gas:

$$p = \frac{RT}{v-b} - \frac{a}{v}$$

$$h = h(v,T) = u + pv = u_0 + c_v T - \frac{a}{v} + pv = u_0 + c_v T - \frac{a}{v} + \frac{RTv}{v-b} - \frac{a}{v}$$

$$= u_0 + c_v T - \frac{2a}{v} + \frac{RTv}{v-b}$$

$$= u_0 + c_v T - \frac{a}{v} + pv$$

c) $$\mu = \left(\frac{\partial T}{\partial p}\right)_h$$

$$\left(\frac{\partial T}{\partial p}\right)_h \left(\frac{\partial p}{\partial h}\right)_T \left(\frac{\partial h}{\partial T}\right)_p = -1$$

$= c_p$

$$\frac{\partial v}{\partial p} = -\kappa v$$

$$\mu = \left(\frac{\partial T}{\partial p}\right)_h = -\frac{1}{c_p}\left(\frac{\partial h}{\partial p}\right)_T = -\frac{1}{c_p}\left(\frac{a}{v^2}\left(\frac{\partial v}{\partial p}\right)_T + v + p\left(\frac{\partial v}{\partial p}\right)_T\right) = -\frac{1}{c_p}\left(\frac{-a\kappa}{v} + v - pv\kappa\right)$$

$$= -\frac{1}{c_p}\left(-\frac{a\kappa}{v} + v - \frac{RTv\kappa}{v-b} + \frac{a\kappa v}{v^2}\right)$$

$$= \frac{RTv\kappa}{c_p(v-b)} - \frac{v}{c_p}$$

d) $$\kappa = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

$$= -\frac{1}{v}\left(\frac{\partial p}{\partial v}\right)_T^{-1}$$

$$= -\frac{1}{v}\left(-\frac{RT}{(v-b)^2} + \frac{2a}{v^3}\right)^{-1}$$

$$= \left(\frac{RTv}{(v-b)^2} - \frac{2a}{v^2}\right)^{-1}$$

e) $a = b = 0 \Rightarrow \kappa = \left(\frac{RT}{v}\right)^{-1} = \frac{v}{RT}$

$$\mu = \frac{RTv\,v}{RTv\,c_p} - \frac{v}{c_p} = 0$$