Exercise 5-10

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$n = 2$ kmol

ideal gas: $PV = nRT$, $Pv = RT$

reversible: $W = PdV$

Ideal monatomic gas: $c_v = \frac{3}{2} R$

$(T_1, V_1) = (300\,K, 2\,m^3)$

$(T_2, V_2) = (400\,K, 2\,m^3)$

$(T_3, V_3) = (400\,K, 10\,m^3)$

$(T_4, V_4) = (300\,K, 10\,m^3)$

isochore: $dQ = n \cdot \left(\frac{1}{n} \left(\frac{\partial Q}{\partial T} \right)_v \right) dT$

$$Q = n \cdot c_v \int_{T_1}^{T_2} dT$$

$$Q_{12} = n c_v (400 - 300) = 2 \cdot \frac{3}{2} \cdot 8{,}314 \cdot 10^3 \cdot 100 = 2{,}49 \cdot 10^6 J$$

$$Q_{23} = W_{23} = \int_{V_2}^{V_3} P dV = nRT \int_{V_2}^{V_3} \frac{dV}{V} = 2 \cdot 8{,}314 \cdot 10^3 \cdot 400 \cdot \ln\left(\frac{10}{2}\right) = 1.07 \cdot 10^7 J$$

$$Q_{34} = n c_v (300 - 400) = -2{,}49 \cdot 10^6 J = -Q_{12}$$

$$Q_{41} = W_{41} = nRT \ln\left(\frac{V_1}{V_4}\right) = 2 \cdot 8{,}314 \cdot 10^3 \cdot 300 \cdot \ln\left(\frac{2}{10}\right) = -8.03 \cdot 10^6 J$$

$$Q_{tot} = Q_{23} + Q_{41} + Q_{12} + Q_{34} = 1.07 \cdot 10^7 - 8.03 \cdot 10^6 = 2.68 \cdot 10^6 J$$

$$W_{cycle} = Q_{tot} = 2.68 \cdot 10^6 J$$

(because, in total, $dT = 0 \Rightarrow dU = 0 \Rightarrow dQ = dW$)

$u = u(T)$ for an ideal gas

$$\eta = \frac{W}{Q_2} = \frac{W}{Q_{in}} = \frac{W}{Q_{12} + Q_{23}} = \frac{2.68 \cdot 10^6}{2.49 \cdot 10^6 + 1.07 \cdot 10^7} = 0.203$$

positive energies/heats are inputs of the heat engine