

a)

~~$$du = T ds - P dv$$~~

$$ds = \frac{dq}{dT} = \frac{C_v dT}{T}$$

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$$C_v = T \left(\frac{ds}{dT} \right) = T \left(\frac{\partial s}{\partial T} \right)_v$$

b)

$$\frac{ds}{dT} = \frac{C_v}{T} = \frac{aT + bT^3}{T} = a + bT^2$$

$$ds = (a + bT^2) dT$$

$$s(T) = \int ds = \int (a + bT^2) dT$$

$$= aT + \frac{1}{3}bT^3 + s_0$$

integration constant