Exercise 6-11

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$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p$$

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - \frac{RT}{v-b} + \frac{a}{v^2}$$

Van der Waals gas:

$$\left(p + \frac{a}{v^2}\right)(v-b) = RT$$

$$p = \frac{RT}{v-b} - \frac{a}{v^2}$$

$$= \frac{RT}{v-b} - \frac{RT}{v-b} + \frac{a}{v^2} = \frac{a}{v^2}$$

$$\cancel{\left(\frac{\partial u}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_u \left(\frac{\partial T}{\partial u}\right)_v = -1}$$

$$\cancel{-\left(\frac{\partial u}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_u = \left(\frac{\partial u}{\partial T}\right)_v = c_v}$$

To show: $c_v = c_v(T)$

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial\left(\frac{RT}{(v-b)} - \frac{a}{v^2}\right)}{\partial T}\right)_v = \frac{R}{v-b}$$

$$c_v = \left(\frac{dq}{dT}\right)_v$$

$$\left(\frac{\partial u}{\partial v}\right)_T = \frac{a}{v^2}$$

$$\partial u = \frac{a}{v^2}\,\partial T$$

$$u = \frac{1}{-1}\,\frac{1}{v}\,a + u_0(T) = -\frac{a}{v} + u_0(T)$$

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v = \left(\frac{\partial u_0(T)}{\partial T}\right)_v = c_v(T)$$