Exercise 7-8

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mass	m	mass	m
temperature	T_1	temperature	T_2

adiabatic : $dQ=0$

Pressure constant: $dP=0$

$h = u + Pv$

$dh = du + d(Pv)$

$dh = Tds - Pdv + Pdv + vdP$ $\qquad dh = c_p\, dT$

$= Tds + vdP$

$= Tds$

$\Downarrow$

$c_p\, dT = Tds$

$ds = \frac{c_p}{T}\, dT$

$\Delta S = c_p \ln\left(\frac{T_f}{T_i}\right)$

$\Delta S = m c_p \ln\left(\frac{T_f}{T_i}\right)$

$dQ_1 + dQ_2 = 0$

$m c_p (T_f - T_1) + m c_p (T_f - T_2) = 0$

$T_f = \frac{T_1 + T_2}{2}$

$\Delta S_{universe} = \Delta S_1 + \Delta S_2$

$T_f = \frac{T_1+T_2}{2}$, as the amount of mass is equal for quantity 1 and quantity 2

$$\Delta S_{universe} = m c_p \left(\ln\left(\frac{T_1+T_2}{2} \cdot \frac{1}{T_1}\right) + \ln\left(\frac{T_1+T_2}{2} \cdot \frac{1}{T_2}\right)\right)$$

$$= m c_p \ln\left(\frac{(T_1+T_2)^2}{4 T_1 T_2}\right)$$

$$= m c_p \ln\left(\left(\frac{T_1+T_2}{2\sqrt{T_1 T_2}}\right)^2\right)$$

$$= 2 m c_p \ln\left(\frac{T_1+T_2}{2\sqrt{T_1 T_2}}\right)$$

$\square$

$\ln\left(\frac{T_1+T_2}{2\sqrt{T_1T_2}}\right) > 0$

$\frac{T_1+T_2}{2\sqrt{T_1T_2}} > 1$

$T_1 + T_2 > 2\sqrt{T_1T_2}$

$T_1^2 + T_2^2 + 2T_1T_2 > 4T_1T_2$

$T_1^2 + T_2^2 > 2T_1T_2$

$T_1^2 + T_2^2 - 2T_1T_2 > 0$

$(T_1 - T_2)^2 > 0$

which holds as long as $T_1 \neq T_2$