Exercise 8-1

wednesday 20 mar 2020 20:44

ideal gas: $PV = nRT \rightarrow P = \frac{nRT}{V}$

Van der Waals gas $P = \frac{nRT}{V - nb} - a\frac{n^2}{V^2}$

reversible isothermal compression

$\downarrow$

$dT = 0$

write down, using $S = S(T, v)$

$$dS = \left(\frac{\partial S}{\partial T}\right)_v dT + \left(\frac{\partial S}{\partial v}\right)_T dv$$

$$= \left(\frac{\partial S}{\partial v}\right)_T dv$$

Maxwell relation: $\left(\frac{\partial S}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v$

$$\rightarrow S = \int \left(\frac{\partial P}{\partial T}\right)_v dv = \begin{cases} \int \frac{R}{v} dv = R \ln\left(\frac{v_f}{v_i}\right) \\ \int \frac{R}{v - b} dv = R \ln\left(\frac{v_f - b}{v_i - b}\right) \end{cases}$$

$v_f < v_i$, or $\frac{v_f}{v_i} < 1$, as this concerns a compression

subtracting a constant from numerator and denominator makes the smaller term (v_f) more dominant. Thus, $\frac{v_f}{v_i} > \frac{v_f - b}{v_i - b}$

and $\ln\left(\frac{v_f}{v_i}\right) > \ln\left(\frac{v_f - b}{v_i - b}\right)$

Now, because both logarithms are less than one, they are both negative. This implies that the larger one is less negative, and thus has a smaller magnitude.

Thus, the Van der Waals gas has the greatest decrease (and thus change) in entropy