Exercise 8-10

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to prove: $C_p = T\left(\frac{\partial s}{\partial T}\right)_p$

and $\left(\frac{\partial C_p}{\partial p}\right)_T = -T\left(\frac{\partial^2 v}{\partial T^2}\right)_p$

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p$$

$$dh = T\,ds + v\,dP$$

$$= \left(\frac{\partial H}{\partial s}\right)_p ds + v\,dP$$

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p \overset{dP=0}{=} \left(\frac{\partial H}{\partial s}\right)_p \left(\frac{\partial s}{\partial T}\right)_p$$

$$= T\left(\frac{\partial s}{\partial T}\right)_p$$

~~to prove: $\left(\frac{\partial s}{\partial T}\right)_p = -\left(\frac{\partial^2 v}{\partial T^2}\right)_p$~~

~~$s = -\frac{\partial v}{\partial T}$~~

$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp$$

constant T $\quad = \left(\frac{\partial s}{\partial p}\right)_T dp$

$$= -\left(\frac{\partial v}{\partial T}\right)_p dp$$

$$\left(\frac{\partial C_p}{\partial p}\right)_T = -T\left(\frac{\partial}{\partial T}\left(\frac{\partial v}{\partial T}\right)_p\right)_p$$

$$= -T\left(\frac{\partial^2 v}{\partial T^2}\right)_p$$

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b) $\left(\frac{\partial C_p}{\partial p}\right)_T = -T\left(\frac{\partial^2 v}{\partial T^2}\right)_p$

$$= -T\left(\frac{\partial^2 \frac{RT}{p}}{\partial T^2}\right)_p$$

$$= -T \cdot 0$$

$= 0$, thus C_p does not depend on P at constant T.

Then, C_p also cannot depend on v at constant T, as that would imply that C_p depends on p, as v depends on P