Exercise 8-8

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a) van der Waals gas: $P = \frac{RT}{v-b} - \frac{a}{v^2}$

$df = -s\,dT - P\,dv$

isothermal: $dT = 0$

$\Downarrow$

$df = -P\,dv$

$$df = \int_{v_1}^{v_2} -\frac{RT}{v-b} + \frac{a}{v^2}\,dv$$

$$= -RT \ln\left(\frac{v_2-b}{v_1-b}\right) - \left(\frac{a}{v_2} - \frac{a}{v_1}\right)$$

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b)

$\Delta u = ?$

$dU = T\,ds - P\,dV$

$s = s(v,T)$

~~$s = -\left(\frac{\partial F}{\partial T}\right)_v = -R \ln\left(\frac{v_2-b}{v_1-b}\right)$~~

$c_v = T\left(\frac{\partial s}{\partial T}\right)_v$

$$ds = \left(\frac{\partial s}{\partial v}\right)_T dv + \left(\frac{\partial s}{\partial T}\right)_v dT$$

$$= \left(\frac{\partial s}{\partial v}\right)_T dv + \frac{c_v}{T}\,dT$$

$$= \left(\frac{\partial P}{\partial T}\right)_v dv + \frac{c_v}{T}\,dT = \frac{R}{v-b}\,dv + \frac{c_v}{T}\,dT$$

↑ maxwell relation

$$du = T\,ds - P\,dv$$

$$= \frac{RT}{v-b}\,dv + \frac{c_v}{T}\,dT - P\,dv$$

$$= \frac{RT}{v-b}\,dv + \frac{c_v}{T}\,dT - \frac{RT}{v-b}\,dv + \frac{a}{v^2}\,dv$$

$$= \frac{a}{v^2}\,dv$$

↑ isothermal → $dT = 0$

$$u = -a\left(\frac{1}{v_2} - \frac{1}{v_1}\right)$$

$$= a\left(\frac{1}{v_1} - \frac{1}{v_2}\right)$$