

$$dU = Tds - PdV + \mu dn$$

a) U is a homogeneous function (multiplying all arguments of U is the same as multiplying U by the same number)
 Therefore, by Euler's theorem:

$$U = S \left(\frac{\partial U}{\partial S} \right)_{V,n} + V \left(\frac{\partial U}{\partial V} \right)_{S,n} + n \left(\frac{\partial U}{\partial n} \right)_{S,V}$$

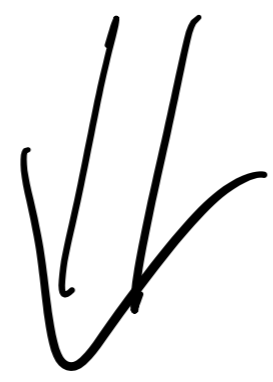
$$= ST - VP + n\mu$$

$$= TS - PV + \mu n$$

$$G = U - TS + VP$$

$$dU = Tds - PdV + \mu dn$$

$$\begin{aligned} dG &= Tds - PdV + \mu dn - Tds - SdT + VdP + PdV \\ &= \mu dn - SdT + VdP \end{aligned}$$



$$(dG)_{T,P} = \mu dn$$

$$\left(\frac{\partial G}{\partial n} \right)_{T,P} = \mu$$

$$F = U - TS$$

$$\begin{aligned} dF &= Tds - PdV + \mu dn - Tds - SdT \\ &= \mu dn - PdV - SdT \end{aligned}$$

$$(dF)_{V,T} = \mu dn$$

$$\mu = \left(\frac{\partial F}{\partial n} \right)_{T,V}$$