

$$\begin{aligned}
 a) \quad dG &= dU - SdT - TdS + VdP + PdV \\
 &= TdS - PdV + \mu dn - SdT - TdS + VdP + PdV \\
 &= -SdT + VdP + \mu dn
 \end{aligned}$$

$$b) \quad P = \frac{nRT}{V-nb} - \frac{an^2}{V^2}$$

$$G = \int \underbrace{-SdT}_{\text{fixed temperature}} + \int VdP + \int \underbrace{\mu dn}_{\text{fixed amount of material}} = \int VdP = PV - \int PdV$$

$$= \frac{nRTV}{V-nb} - \frac{an^2}{V} - \int \left(\frac{nRT}{V-nb} - \frac{an^2}{V^2} \right) dV$$

$$= \frac{nRTV}{V-nb} - \frac{an^2}{V} - nRT \ln(V-nb) - \frac{an^2}{V} + C'$$

$$= \frac{nRT(V-nb+nb)}{V-nb} - \frac{2an^2}{V} - nRT \ln(V-nb) + C'$$

$$= -nRT \ln(V-nb) + \frac{n^2 b RT}{V-nb} - \frac{2n^2 a}{V} + nRT + C'$$

$$= -nRT \ln(V-nb) + \frac{n^2 b RT}{V-nb} - \frac{2n^2 a}{V} + C(T)$$