

$$T \left(\frac{\partial S}{\partial T} \right)_V = C_V$$

$$\left(\frac{\partial S}{\partial T} \right)_V = \frac{C_V}{T} = \frac{bT^\alpha}{T} = bT^{\alpha-1}$$

$$dS = \int bT^{\alpha-1} dT$$

$$\Delta S = S - S_0 = \frac{b}{\alpha} (T^\alpha - T_0^\alpha)$$

$\Delta S = 0$ implies $T^\alpha \rightarrow T_0^\alpha$, which, as $T > T_0$, implies that $\alpha > 0$

$$C_V = aT + bT^3$$

$$dS = \frac{C_V}{T} dT$$

$$\Delta S = \int_{T_0}^T \frac{aT + bT^3}{T} dT$$

$$= \int_{T_0}^T a + bT^2 dT$$

$$= a(T - T_0) + \frac{b}{3} (T^3 - T_0^3)$$