

$$PV + f(V) = AU$$

$$C_V = \left( \frac{\partial u}{\partial T} \right)_V$$

$$C_V = \left( \frac{\partial}{\partial T} \left( \frac{PV + f(V)}{A} \right) \right)_V$$

$$= \frac{V}{A} \left( \frac{\partial p}{\partial T} \right)_V + \frac{p}{A} \left( \frac{\partial V}{\partial T} \right)_V + \frac{1}{A} \left( \frac{\partial f(V)}{\partial T} \right)_V$$

$$= \frac{V}{A} \left( \frac{\partial p}{\partial T} \right)_V = \frac{V}{A} \left( \frac{\partial S}{\partial V} \right)_T$$

Maxwell relation  $\rightarrow$

$$\lim_{T \rightarrow 0} C_V = \lim_{T \rightarrow 0} \frac{V}{A} \left( \frac{\partial S}{\partial V} \right)_T = 0, \text{ as the change of } S \text{ goes to zero as } T \text{ goes to zero. (by the 3}^{\text{rd}} \text{ law)}$$