Exercise 12-12

woensdag 20 mei 2020 20:47

Consider a gas consisting of one kilomole of helium atoms at standard temperature and pressure. Calculate the degeneracy $g(\epsilon)$ for the energy level $\epsilon = \frac{3}{2}kT$ (take $\gamma_s = 1$). What is the approximate ratio of $g(\epsilon)$ to the number of atoms N?

$$g(\varepsilon)d\varepsilon = \gamma_s \frac{4\sqrt{2}\,\pi V}{h^3} m^{\frac{3}{2}} \varepsilon^{\frac{1}{2}} d\varepsilon$$

$$g(\varepsilon) = \frac{4\sqrt{2}\,\pi V}{h^3} m^{\frac{3}{2}} \left(\frac{3}{2} k_B T\right)^{\frac{1}{2}}$$

$$pV = N k_B T \Rightarrow V = \frac{N k_B T}{p}$$

$$m = \frac{M_w n}{N_A}$$

(m is the mass of one particle)

$$\rightarrow g(\varepsilon) = \frac{4\sqrt{2}\,\pi N k_B T\, M_w^{\frac{3}{2}} \times (10^3 mol)^{\frac{3}{2}}}{p h^3} \left(\frac{3}{2} k_B T\right)^{\frac{1}{2}}$$

$$\frac{g(\varepsilon)}{N} = \frac{4\sqrt{3}\,\pi\sqrt{\pi}\,(k_B T M_w)^{\frac{3}{2}}}{p h^3 N_A^{\frac{3}{2}}} \times 10^3 \sqrt{10^3}$$

$p = 10^5\ N\,m^{-2}$

$N_A = 6 \cdot 10^{23}\ mol^{-1}$

$T = 300\ K$

$h = 6.626069\,57 \times 10^{-34}\ J_S$

$M_w = 4\ g/mol = 0.004\ kg/mol$