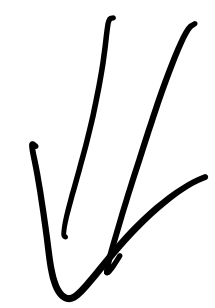


to prove:  $f(xy) = f(x) + f(y)$



$$f(x) = C \ln(x)$$

$$\frac{\partial}{\partial x} f(xy) = y \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} f(x)$$

$$\frac{\partial}{\partial y} f(xy) = x \frac{\partial f}{\partial y} = \frac{\partial}{\partial y} f(y)$$

$$\frac{\partial f(xy)}{\partial x} = y \frac{\partial f}{\partial x} = \frac{\partial f(x)}{\partial x}$$

$$\frac{\partial f(xy)}{\partial y} = x \frac{\partial f}{\partial y} = \frac{\partial f(y)}{\partial y}$$

$$\frac{\partial f(x)}{\partial x} = \frac{\partial f(y)}{\partial y}$$

$$x \frac{\partial f(x)}{\partial x} = y \frac{\partial f(y)}{\partial y} = \text{constant}$$

↑  
function of x only

↑  
function of y only

$$f'(x) = \frac{\alpha}{x} \Rightarrow f(x) = \alpha \ln x + C'$$

$$= C \ln x$$