Exercise S8

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$$U = \mu_B B N \cdot \begin{cases} +1 & \text{for spin down} \\ -1 & \text{for spin up} \end{cases}$$

$$U = -\mu_B B N^{\uparrow} + \mu_B B N^{\downarrow}$$

$$= -\mu_B B N^{\uparrow} + \mu_B B (N - N^{\uparrow})$$

$$= \mu_B B N - 2\mu_B B N^{\uparrow}$$

$$= \mu_B B N (1 - 2\phi) \qquad \phi = \frac{N^{\uparrow}}{N}$$

$$S = k_B \ln W$$

$$= k_B \ln \binom{N}{N^{\uparrow}}$$

$$= k_B \left(\ln N! - \ln N^{\uparrow}! - \ln (N - N^{\uparrow})! \right)$$

$$= k_B \left(N \ln N - N - N^{\uparrow} \ln N^{\uparrow} + N^{\uparrow} - (N - N^{\uparrow}) \ln (N - N^{\uparrow}) + (N - N^{\uparrow}) \right)$$

$$= k_B \left((N - N^{\uparrow} + N^{\uparrow}) \ln N - N^{\uparrow} \ln N^{\uparrow} - (N - N^{\uparrow}) \ln (N - N^{\uparrow}) \right)$$

$$= k_B \left(-(N - N^{\uparrow}) \ln \frac{1}{N} - N^{\uparrow} \ln \frac{1}{N} - N^{\uparrow} \ln N^{\uparrow} - (N - N^{\uparrow}) \ln (N - N^{\uparrow}) \right)$$

$$= k_B \left(-N^{\uparrow} \ln \left(\frac{N^{\uparrow}}{N} \right) - (N - N^{\uparrow}) \ln \left(\frac{N - N^{\uparrow}}{N} \right) \right)$$

$$= k_B N \left(-\frac{N^{\uparrow}}{N} \ln \left(\frac{N^{\uparrow}}{N} \right) - \frac{N - N^{\uparrow}}{N} \ln \left(\frac{N - N^{\uparrow}}{N} \right) \right)$$

$$= -k_B N \left(\frac{N^{\uparrow}}{N} \ln \left(\frac{N^{\uparrow}}{N} \right) + \frac{N - N^{\uparrow}}{N} \ln \left(\frac{N - N^{\uparrow}}{N} \right) \right)$$

$$= -k_B \left(\phi \ln \phi + (1 - \phi) \ln (1 - \phi) \right) \qquad \phi = \frac{N^{\uparrow}}{N}$$