Exercise S9

woensdag 20 mei 2020 20:47

T is kept constant
N is kept constant
V is kept constant

$\Big\}$ (N, V, T) system $\Rightarrow$ Helmholtz function

$$F = U - TS$$

$$\text{equilibrium} \Rightarrow \frac{\partial F}{\partial \phi} = 0$$

$$F = U - TS = \mu_B B N (1 - 2\phi) + T k_B N \left(\phi \ln \phi + (1-\phi) \ln (1-\phi)\right)$$

$$\frac{\partial F}{\partial \phi} = -2\mu_B B N + k_B T N \left(\ln \phi + 1 - \ln(1-\phi) - 1\right)$$

chain rule, product rule +2

$$0 = -2\mu_B B + k_B T \left(\ln \phi - \ln (1-\phi)\right)$$

$$\frac{2\mu_B B}{k_B T} = \ln\left(\frac{\phi}{1-\phi}\right)$$

$$\frac{\phi}{1-\phi} = e^{\frac{2\mu_B B}{k_B T}}$$

$$\phi = (1-\phi)\, e^{\frac{2\mu_B B}{k_B T}}$$

$$\phi\left(1 + e^{\frac{2\mu_B B}{k_B T}}\right) = e^{\frac{2\mu_B B}{k_B T}}$$

$$\phi = \frac{e^{\frac{2\mu_B B}{k_B T}}}{1 + e^{\frac{2\mu_B B}{k_B T}}}$$

---

b)

$$M = \mu_B (N_\uparrow - N_\downarrow)$$

$$= \mu_B \left(N_\uparrow - (N - N_\uparrow)\right)$$

$$= \mu_B (2N_\uparrow - N)$$

$$= \mu_B N (2\phi - 1)$$

$$\frac{M}{\mu_B N} = 2\phi - 1 = \tanh\left(\frac{\mu_B B}{k_B T}\right)$$

$$\phi = \frac{e^{\frac{2\mu_B B}{k_B T}}}{1 + e^{\frac{2\mu_B B}{k_B T}}}$$

$$= \frac{e^{\frac{\mu_B B}{k_B T}}}{e^{\frac{\mu_B B}{k_B T}} + e^{-\frac{\mu_B B}{k_B T}}}$$