Exercise 13.1

vrijdag 29 mei 2020 11:28

We want to maximize $A(x,y) = xy$

under constraint $g(x,y) = x + y - 8 = 0$

$= \text{constant}$

$dg = \frac{\partial g}{\partial x} dx + \frac{\partial g}{\partial y} dy = 0$

$$dA = \frac{\partial A}{\partial x} dx + \frac{\partial A}{\partial y} dy = 0$$

$y\,dx + x\,dy = 0$

$1dx + 1dy = 0$

$dx + dy = 0$

$dy = -dx$

$y\,dx - x\,dx = 0$

$(y - x)\,dx = 0$

$y - x = 0$

$y = x$

$2x = 8 \rightarrow x = 4$

$A(xy) = xy = 4 \cdot 4 = 16$

$$\frac{\partial A}{\partial x} + \alpha \frac{\partial g}{\partial x} = 0 \qquad \frac{\partial A}{\partial y} + \alpha \frac{\partial g}{\partial y} = 0$$

$y + \alpha = 0 \qquad x + \alpha = 0$

$y = -\alpha \qquad x = -\alpha$

$-\alpha - \alpha - 8 = 0$

$\alpha = -4$

$x = y = 4$

$A = xy = 4 \cdot 4 = 16$

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b)

$$g(x,y) = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$$

$$A(x,y) = xy$$

$$\frac{\partial f}{\partial x} + \alpha \frac{\partial g}{\partial x} = 0 \qquad \frac{\partial f}{\partial y} + \alpha \frac{\partial g}{\partial y} = 0$$

$y + 2\alpha a^2 x = 0 \qquad x + 2\alpha b^2 y = 0$

$y = -2\alpha a^2 \cdot -2\alpha b^2 y \qquad x = -2\alpha b^2 y$

$1 = 4\alpha a^2 b^2$

$$\alpha = \frac{1}{4a^2b^2}$$

$$y + \frac{2a^2 x}{4a^2b^2} = 0$$

$$y = -\frac{x}{2b^2}$$

$$\frac{x^2}{a^2} + \frac{\left(-\frac{x}{2b^2}\right)^2}{b^2} - 1 = 0$$

$$\frac{x^2}{a^2} + \frac{x^2}{4b^6} = 1$$

$$x^2\left(\frac{1}{a^2} + \frac{1}{4b^6}\right) = 1$$

$$x^2 = \frac{1}{\frac{1}{a^2} + \frac{1}{4b^6}} = \frac{1}{\frac{4b^6 + a^2}{4a^2b^6}} = \frac{4a^2b^6}{4b^6 + a^2}$$

$$x = \pm 2ab^3 \times \frac{1}{\sqrt{4b^6 + a^2}}$$

$$y = -\frac{x}{2b^2}$$

take + variant

$$A = xy =$$