Lecture notes

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occupation numbers

state $i(j) \Rightarrow N_{i(j)}$

level $j \Rightarrow N_j = \sum_{i(j)=1}^{g_j} N_{i(j)}$

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microcanonical = isolated system

$$\left.\begin{aligned} N_j &= \sum_{i(j)=1}^{g_j} N_{i(j)} \\ N &= \sum_{j=1}^{n} N_j \\ U &= \sum_{j=1}^{n} N_j E_j \end{aligned}\right\} (N, V, U)$$

$$W_B \underbrace{(N_1, N_2, \ldots, N_n)}_{\text{macrostate}}$$

chose N_1 particles from N

divide N_1 particles over g_1 states

particles : distinguishable

Statistical postulate: $p = \frac{W_B}{\Omega_B}$

largest term W_B = most probable

most probable distribution

$$\frac{\partial S_B}{N_i} + \alpha + \beta E_i = 0$$

$$N_j = g_j e^{\frac{\alpha + \beta E_j}{k_B}}$$

$$\left.\begin{aligned} S_B &= k_B N \ln(N) - N\alpha - \beta U \\ \left(\frac{\partial S}{\partial U}\right)_{N,V} &= \frac{1}{T} \end{aligned}\right\} \beta = -\frac{1}{T}$$

$$f = \frac{N_j}{g_j} = \frac{N}{Z} \exp\left(-\frac{E_j}{k_B T}\right)$$

$$Z = \sum_{j=1}^{n} g_j \exp\left(-\frac{E_j}{k_B T}\right)$$

$$P_j = \frac{N_j}{N} = \frac{g_j}{Z} \exp\left(-\frac{E_j}{k_B T}\right)$$ probability that a level j is occupied

$$P(E)\,dE = \frac{g(E)}{Z} \exp\left(-\frac{E}{k_B T}\right) dE$$

$$Z = \int_0^\infty dE\, g(E) \exp\left(-\frac{E}{k_B T}\right)$$

$$\left(\frac{\partial S}{\partial N}\right)_{U,V} = -\frac{\mu}{T} \Longrightarrow \mu = -k_B T \ln(Z)$$

Spin $\frac{1}{2}$ - paramagnet $\quad Z = 2\cosh\left(\frac{\mu_B B}{k_B T}\right) = Z(T)$