Exercise 14.7

vrijdag 29 mei 2020 11:29

$$Z = a V T^4$$

$$U, P, S = ?$$

$$\frac{U}{N} = -\frac{\partial \ln Z}{\partial \beta} = -\frac{\partial}{\partial \frac{1}{k_B T}} \ln Z = -\frac{1}{Z} \frac{\partial}{\partial \frac{1}{k_B T}} Z = \frac{k_B T^2}{Z} \frac{\partial Z}{\partial T} = 4 k_B T$$

$$S = \frac{U}{T} + N k_B \left(\ln(Z) - \ln(N) + 1 \right) = 4 k_B N + N k_B \left(\ln \frac{a V T^4}{N} + 1 \right) = N k_B \left(5 \ln \left(\frac{a V T^4}{N} \right) \right)$$

$$F = -N k_B T \ln \frac{Z}{N} + N k_B T$$

$$P = -\left(\frac{\partial F}{\partial V} \right)_{N,T} = N k_B T \; \frac{1}{\frac{Z}{N}} \frac{\partial Z / \partial V}{N} = \frac{N k_B T}{V}$$