Exercise S18

vrijdag 29 mei 2020 11:30

classical limit : $N_j \ll g_j$

$$W_{FD} = \prod_{j=1}^{n} \frac{g_j!}{N_j!\,(g_j-N_j)!} = \prod_{j=1}^{n} \frac{g_j!}{N_j!\,(g_j-N_j)!} = \prod_{j=1}^{n} \frac{g_j\,(g_j-1)\cdots(g_j-N_j+1)\,(g_j-N_j)!}{N_j!\,(g_j-N_j)!} = \prod_{j=1}^{n} \frac{g_j\,(g_j-1)\cdots(g_j-N_j+1)}{N_j!} \approx \prod_{j=1}^{n} \frac{g_j^{N_j}}{N_j!}$$

$$W_{BE} = \prod_{j=1}^{n} \frac{(N_j+g_j-1)!}{N_j!\,(g_j-1)!} = \prod_{j=1}^{n} \frac{(N_j+g_j-1)(N_j+g_j-2)\cdots \overset{=(N_j+g_j-N_j)}{g_j}(g_j-1)!}{N_j!\,(g_j-1)!} = \prod_{j=1}^{n} \frac{(N_j+g_j-1)(N_j+g_j-2)\cdots g_j}{N_j!} \approx \prod_{j=1}^{n} \frac{g_j^{N_j}}{N_j!}$$

which implies $W_{FD} = W_{BE}$ $(= W_{MB})$